“Topology and physics: from quantum Hall effect to planetary waves”

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In the year 1982, soon after the discovery of the quantum Hall effect, four scientists: Thouless, Kohmoto, Nightingale and de Nijs explained it in terms of topology by introducing the concept of the topological invariant of a band structure. Mathematically, topology deals with such properties of various objects which do not change under continuous transformations. ("Topology is a science of objects made out of rubber." - M. Szopa) In physics, topological properties do not change during adiabatic evolution of parameters - for instance, under a slow variation of a magnetic field. As it turned out, quantized Hall conductance is nothing else but a manifestation of the quantized topological invariant of the space of quantum states. In non-interacting systems the principle of the bulk-edge correspondence tells us that if the invariant is non-zero, the edges between two different topological phases host boundary modes, resistant to local perturbations. This topological protection is one of the main reasons behind the vigorous development of topological physics in recent years.

In this talk we shall explore the non-trivial topology, manifesting physically through the formation of boundary states, in three different systems.
The edge modes of the quantum Hall effect form one-dimensional channels, in which the electronic charge flows without scattering and in one direction only. The non-trivial topology is responsible for the quantized values of the Hall conductance. In one-dimensional topological superconductors the boundary states have a Majorana nature - they are their own antiparticles - and their statistics is neither fermionic nor bosonic. Since one state resides on both ends of the superconductor, these Majorana states are also protected from local perturbation and can serve as a basis for error-resistant topological qubits. For the finish we shall look at a fully classical example of planetary waves. At the equator the changing sign of the Coriolis force divides the Earth in two topological phases, the north and south hemispheres. This leads to the formation of equatorial (so-called Kelvin) waves, propagating unidirectionally and with frequency lower than that of the usual gravity waves. These three examples illustrate the power of topological arguments applied to physics, and the variety of phenomena which can be explained thanks to this seemingly abstract branch of mathematics.