Abstract

The first part of this dissertation is focused on the generalization or extension of the results of W. Fechner and E. Gselmann (Publ. Math. Debrecen 80/1-2 (2012), 143-154, `https://doi.org/10.5486/PMD.2012.4970`) into new classes of functional equations and determining their solutions. Since no regularity is assumed, it turns out that under some mild assumptions on the parameters involved, the pairs of functions satisfying the new classes of functional equations are polynomial functions and, in some crucial cases, just the usual polynomials. The idea to study these generalized equations was motivated by the growing number of its particular forms studied by some mathematicians.

Several mathematicians attempted to solve equations characterizing the polynomial functions of restricted domains; therefore, in this spirit, this dissertation's second part focuses on finding the local polynomial functions stemming from these new classes of functional equations.

Finally, we develop a robust computer code based on the obtained theoretical results to determine the polynomial solutions of these generalized equations. The primary motivation for writing such a computer code is that solving even simple equations belonging to these classes needs long and tiresome calculations. Therefore, one of the advantages of such a computer code is that it allows us to solve complicated problems quickly, easily, and efficiently. Additionally, the computer code will significantly improve the level of accuracy in calculations. Along with that, there is also the factor of speed. We point out that the computer code will operate with symbolic calculations provided by the Python programming language, which means that it does not contain any numerical or approximate methods, and it yields the exact solutions of the equations considered. We acknowledge that some mathematicians have previously considered using computer codes to solve functional equations. However, In their works, they used Maple as the programming tool to obtain their results which is less flexible in usage and constitutes only a small portion of the academic research community; whereas, in our work, we achieved our results using Python programming language, designed to be an easily readable, highly versatile, general-purpose, open-source, avails robustness and facilitates the deployment of theorems into computational and symbolic frameworks.

Keywords: Functional equations, Polynomial functions, Absolutely convex sets, Algebraic interior, Fréchet operator, Monomial functions, Continuity of monomial functions, Computer assisted methods, Python, Sagemath.