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## Summary of the PhD thesis

# On some properties of bases and $\pi$-bases of topological spaces

This dissertation focuses on two properties considered for bases and $\pi$-bases of topological spaces. The first of the discussed properties, introduced by R. Freese and J. B. Nation in [7], was originally considered for lattices. The second property considered by W. Fleissner and L. Yengulalp in [6] is related to the domain theory introduced by D. Scott in [9]. The results on Freese–Nation properties are presented in Chapters 2–5. They have been published in [1], [4] and [3]. The results for spaces represented by the domain were presented in Chapter 6 and published in [2]. In the first chapter, we recall notions of topology and set theory that we need in the following chapters.

For historical reasons, we begin the second chapter by introducing properties for Boolean algebras. In this chapter we introduce three properties:

- the Freese–Nation property,
- the separative Freese–Nation property,
- the interpolative Freese–Nation property.

For Boolean algebras these properties are equivalent. Then we introduce these three properties for topological spaces. In the next paragraph, we indicate some classes of topological spaces with the Freese–Nation property.

The third chapter focuses on spaces that can be represented as the limit of an inverse system satisfying some conditions. The first paragraph concerns openly generated spaces introduced by E. V. Shchepin in [10]. We show that in the class of compact Hausdorff zero-dimensional spaces the separative Freese–Nation property for the family of all clopen subsets is equivalent to being openly generated space. We show that compact Hausdorff spaces having the separative Freese–Nation property for some base consisted of functionally open sets are openly generated spaces. In the second paragraph, we show an analogous result for the $\pi$-separative Freese–Nation property and skeletally generated spaces. In the last paragraph, we use the open-open game introduced in [5] to prove the theorems from two previous paragraphs concerning openly generated and skeletally generated spaces.

In Chapter 4 we prove that the family of all regularly open subsets of infinite regular space and topology of infinite regular space do not have the separative Freese–Nation property and the Freese–Nation property. This means that if a topological space has a base with the Freese–Nation property, then the extension of that base does not have to have the Freese–Nation property.

The results presented in Chapter 5 concern coabsolute spaces. In the first paragraph, we show that a coabsolute space to a space with $\pi$-separative Freese–Nation property has $\pi$-separative Freese–Nation property. Skeletally Dugundji spaces are analogous to Dugundji spaces, introduced by A. Pełczyński in [8]. In the second paragraph, we prove that each skeletally Dugundji space has the $\pi$-separative Freese–Nation property.

The main results of Chapter 6 are related to the Banach–Mazur game and the Choquet game. We show that spaces for which there exists a winning strategy for Player II in the Banach–Mazur game (the Choquet game) can be characterized as countably $\pi$-domain representable spaces (countably domain representable spaces).

# References


[1] J. Bąk, A. Kucharski, *Topological spaces with the Freese–Nation property*, Annales Mathematicae Silesianae, 33 (2019), 41–54.

[2] J. Bąk, A. Kucharski, *The Banach–Mazur game and domain theory*, Arch. Math., 114 (2020), 51–59.

[3] J. Bąk, A. Kucharski, *An internal characterization of complete regularity*, Mathematica Slovaca, vol. 70, issue 3, (2020), 775–777.

[4] J. Bąk, A. Kucharski, *Topological spaces with the Freese–Nation property II*, arXiv:1904.08902, accepted in Topology Appl.

[5] P. Daniels, K. Kunen, H. Zhou, *On the open-open game*, Fund. Math., 145 (1994), no. 3, 205–220.

[6] W. Fleissner, L. Yengulalp, *When $C_p(X)$ is Domain Representable*, Fund. Math., 223 (1) (2013), 65–81.

[7] R. Freese, J. B. Nation, *Projective lattices*, Pacific Journal of Mathematics, 75 (1975), 93–106.

[8] A. Pełczyński, *Linear extensions, linear averagings, and their applications to linear topological classification of spaces of continuous functions*, Dissert. Math., 58 (1968), 1–89.

[9] D. Scott, *Outline of a mathematical theory of computation*, Technical Monograph PRG-2, November 1970.

[10] E. V. Shchepin, *Topology of limit spaces with uncountable inverse spectra*, Uspekhi Mat. Nauk, 31 (1976), no. 5 (191), 191–226.