Marta Walczyńska, MSc
Istitute of Mathematics
Faculty of Mathematics, Physics and Chemistry
University of Silesia in Katowice

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Summary of the PhD thesis

Applications of category theory to general topology

This dissertation is concerned with some notions of category theory in application to general topology and investigates to what extent the arguments making use of those notions can replace a more traditional approach. One of the generic examples is the notion of Fraisse sequence, whose applications have been presented in papers [3], [4] and [5].

Quite a number of categories consist of algebraic objects: groups, Abelian groups and monoids with group, Abelian group and monoid homomorphisms, respectively. An example of a monoid consisting of topological operations is the Kuratowski monoid dealt with in paper [6].

The dissertation is organized as follows. In subsection 1.2.1 we give some comments on ways of making use of concepts such as a tree, the space of branches, an ultra-metric and so forth.

In section 2 we provide axioms for categories, basic definitions, notations and examples. We draw attention to a particular covariant functor, namely a *diagram*.

At the beginning of section 3 we remind basic ideas concerning the limit of inverse sequence. Then we analyze a proof of Knaster–Reichbach theorem about extendibility of a mapping between two closed, nowhere dense subsets of the Cantor set and its version for scattered compact metric spaces. Subsection 3.3 contains some of the results from paper [7], in which, however without direct reference to category theory, the category of σ-discrete metric spaces with embeddings as morphisms has been investigated.

In section 4 we focus on *a symmetric Cantorval*. Beside the Cantor set and the sum of a finite family of closed intervals it is one of the possible forms the set of subsums of a convergent series with positive terms can take. The findings that have been obtained are published in paper [1] and their comprehensive summary makes up subsection 4.1. There, as well as in subsection 4.2, where we present a generalization of J.von Neumann theorem included in paper [1], we apply so-called *back-and-forth* method. It has proven to be useful in proving properties of universal homogeneous objects, i.e. Fraïssé limits, as substantiated in the subsequent sections of the dissertation.

In section 5 we remind the notions of the amalgamation and the inverse amalgamation property as well as their special cases - *pushout* and *pullback*, respectively. We discuss these notions in two particular categories: **Set** category whose objects are sets and whose arrows are functions and **Top** category whose objects are topological spaces and whose arrows are continuous mappings.

In section 6 we present the main idea of the dissertation, namely Fraïssé sequences: definition, criterion of existence and some properties. In subsection 6.2 we focus on the category $\mathbf{Finset}^{\mathrm{op}}/A$, i.e. a comma-category related to the opposite category of the category **Finset**.

In section 7 we discuss the structure and basic facts concerning Fraïssé limits in the context of inverse sequences. We show a way in which the properties of *projective universality* and *projective homogeneity* combine with the concept of *generic* object (for a given subcategory).

Section 8 comprises comments that will be included in joint papers with the supervisors concerning, among others, a category of finite non-empty sets and its linear version.
We claim here that the Cantor set admits a unique strictly positive probability measure that takes rationa values on clopen sets and satisfies certain homogeneity condition.

References

[1] W. Bielas, Sz. Plewik oraz M. Walczyńska, *On the center of distances*, European Journal of Mathematics, DOI: 10.1007/s40879-017-0199-4 lub arXiv: 1605.03608v2.

[2] W. Kubiś, *Fraïssé sequences: category-theoretic approach to universal homogeneous structures*, Ann. Pure Appl. Logic 165 (2014), 1755–1811.

[3] W. Kubiś, *Injective objects and retracts of Fraïssé limits*, Forum Math. 27 (2015), 807–842.

[4] W. Kubiś, *Banach–Mazur game played in partially ordered sets.* Algebra, logic and number theory, Banach Center Publ., 108, Polish Acad. Sci. Inst. Math., Warsaw, (2016), 151–160.

[5] W. Kubiś oraz A. Kwiatkowska *The Lelek fan and the Poulsen simplex as Fraïssé limits*, RACSAM. (2017).

[6] Sz. Plewik oraz M. Walczyńska, *The Monoid Consisting of Kuratowski Operations*, Journal of Mathematics (2013), 1–9.

[7] Sz. Plewik oraz M. Walczyńska, *On metric σ-discrete spaces*, Algebra, logic and number theory, 239–253, Banach Center Publ., 108, Polish Acad. Sci. Inst. Math., Warsaw, (2016), 239–253.