Abstract

This dissertation consists of the following series of four publications on the application of piecewise deterministic Markov processes (PDMPs) in modelling population dynamics:

[I] P. Klimasara. Revisiting the logistic growth with random disturbances. *Math. Appl. (Warsaw)*, 47(2):177–186, 2019.

[II] P. Klimasara and M. Tyran-Kamińska. A model for random fire induced tree-grass coexistence in savannas. *Math. Appl. (Warsaw)*, 46(1):87–96, 2018.

[III] P. Klimasara and M. Tyran-Kamińska. A model of seasonal savanna dynamics. *SIAM J. Appl. Math.*, 83(1):122–143, 2023.

[IV] P. Klimasara, M. C. Mackey, A. Tomski, and M. Tyran-Kamińska. *Randomly switching evolution equations. Nonlinear Anal. Hybrid Syst.*, 39:Paper No. 100948, 15, 2021.

The research problem, thematically linking the articles that constitute this dissertation, is the analysis of the impact of different types of perturbations on population dynamics models based on PDMPs and the development of formal methods to analyze such models.

After a concise presentation of key concepts – and short mathematical preliminaries – the main part of the dissertation is divided into three sections.

In the first, we present two population models of antagonistic plant groups – grasses and trees – in the savanna. It is conjectured that the lack of transition to another biome, in which one of them dominates, is due to the frequent occurrence of environmental disturbances that cause losses in the biomass of both vegetation types. Among the most important such factors are fires. Most existing mathematical models do not directly account for the random nature of fires – either by assuming their periodic recurrence, or by introducing expressions corresponding to continuous biomass losses into the equations (deterministic models). Furthermore, they often lack analytical results, and are limited to numerical simulations and bifurcation analysis. In paper [I], we study a one-dimensional model in which the logistic growth of tree biomass (assuming that grasses would maximally fill all remaining space) is perturbed at random times – denoting fire outbreaks – by the discrete loss of a random fraction of the accumulated biomass. On the other hand, in paper [II] we analyze a two-dimensional model that considers also grass biomass and its growth rate, but takes the amount of plant biomass loss due to fire as a fixed fraction of the biomass accumulated so far (value at the time of fire outbreak). In both articles, we use PDMPs to describe the model and – by examining *stochastic semigroups* induced

by these processes – we justify mathematically that random fires allow the possibility of (typical for savannas) long-time tree-grass coexistence.

In the next part of the thesis we extend the models from [I] and [II] with additional variables – introducing the populations of herbivores and taking into account the impact of their occurrence on grass and tree biomasses. Moreover, we propose an approach that allows us to take into account a periodic "environmental disturbance" – seasonality (in the case of savanna: switching between dry season and wet season). Its occurrence is very important for many different ecosystems, but unfortunately the inclusion of seasons in mathematical models poses many technical difficulties in their study. In the paper [III], we approach the subject in a new (for this problem) way – by using PDMPs with two types of switching: jumps in phase space (fire-induced losses) at random times and with random severity (biomass damage), and discrete changes in the model dynamics at fixed intervals (i.e. during the change of seasons). In the proposed PDMPs, the periodicity of the additional time variable (counting the time since the last season change) prevents us from examining the convergence of the distributions to a stationary distribution and we instead examine the convergence of time averages for such processes. In particular, we provide conditions for which stationary distributions for grass and tree biomasses (as well as populations of additionally introduced herbivores) exist. These methods can be used also for such processes when more seasons than two are present.

The final part of the thesis – based on the publication [IV] – deals with models in which random environmental disturbances affect all individuals simultaneously. Conducting considerations from a population perspective – in contrary to so-called individual perspective – leads us to models with an infinite-dimensional state space (where states are represented by population density). The study of the corresponding evolution equations poses many formal difficulties – the only results in the literature are determination of moment equations for diffusion processes in randomly switching environment. We extend these results for a broader class of processes – described by *randomly switching stochastic semigroups* – and we study *the mean of such processes at large time.*