



**Juliusz P. Schauder  
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Nicolaus Copernicus University in Toruń

# **Linear iterative functional equations with random-valued functions**

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