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Global central limit theorems for stationary Markov chains

Let $P = P(x, A)$ be a Markov transition probability on a general state space (S, Σ) , with invariant probability m . Let $\Omega := S^{\mathbb{N}}$ be the space of trajectories with σ -algebra $\mathcal{A} := \Sigma^{\otimes \mathbb{N}}$, and let \mathbb{P}_m be the probability on \mathcal{A} of the chain with transition probability P and initial distribution m . By invariance of m , \mathbb{P}_m is shift-invariant on (Ω, \mathcal{A}) . Let X_n be the projection of Ω on the n th coordinate. Then (X_n) on $(\Omega, \mathcal{A}, \mathbb{P}_m)$ is a stationary Markov chain with state space S .

We assume m ergodic for P , so the chain is ergodic too, i.e. the shift on Ω is ergodic.

We say that a real centered $f \in L_2(m)$ satisfies the *annealed CLT* if in (Ω, \mathbb{P}_m) we have

$$\frac{1}{\sqrt{n}} \sum_{k=1}^n f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma^2), \quad \text{where } \mathcal{N}(0, 0) := \delta_0.$$

We say that a real centered $0 \neq f \in L_2(m)$ satisfies the *L_2 -normalized CLT* if

$$\frac{1}{\sigma_n(f)} \sum_{k=1}^n f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1),$$

where $\sigma_n(f) := \|\sum_{k=1}^n f(X_k)\|_2 > 0$ for large n .

We study conditions which yield that for every centered $0 \neq f \in L_2(m)$ a non-degenerate ($\sigma^2 > 0$) annealed CLT and an L_2 -normalized CLT hold.

Joint work with Christophe Cuny