Załącznik do ogłoszenia Dyrektora
Instytutu Matematyki z dnia
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THE SUBSTANTIVE SCOPE OF THE INTERVIEW FOR ADMISSION TO THE DOCTORAL SCHOOL OF THE UNIVERSITY OF SILESIA IN THE DISCIPLINE OF MATHEMATICS

English version

1. Algebra

- Group theory: groups, subgroups, group homomorphisms, normal subgroups, quotient groups, isomorphism theorems, classical examples of groups, permutation groups.
- Ring theory: rings, subrings, ideals, principal ideals, prime and maximal ideals, integral domains, Euclidean rings and principal ideal domains, unique factorization domains, local rings and localization at an ideal, polynomial rings.
- Field theory: fields and subfields, field extensions, finite and algebraic extensions, algebraic elements, minimal polynomial of an algebraic element, splitting field of a polynomial, algebraically closed fields, finite fields, multiplicative group of a finite field.
- Linear algebra: modules and submodules, quotient modules, module homomorphisms, free modules and vector spaces, basis, rank of a module and dimension of a vector space, projective modules, eigenvalues and eigenvectors, determinant and characteristic polynomial.
- Homological algebra: module complexes and exact sequences, chain modules, boundary homomorphisms, boundary and cycle modules, homology modules.

2. Functional analysis

- Normed spaces and Banach spaces, examples.
- Linear operators in normed spaces, continuous linear functionals, norm in the space $L(X, Y)$. Dual normed space.
- The Hahn-Banach theorem and its consequences.
- Open mapping theorem, closed graph theorem. Banach-Steinhaus theorem.
- Inner product spaces and Hilbert spaces, examples. Parallelogram law, Cauchy-Schwarz inequality.
- Orthogonality and orthogonal projection. Orthogonal projection on a convex set. Orthogonal complement.
- The Riesz representation theorem for continuous linear functionals in Hilbert space. Dual space.
- Orthogonal systems and Fourier series in Hilbert spaces. The trigonometric system and its completeness.

3. Computational mathematics

- Polynomial arithmetic: fast multiplication algorithms, polynomial division and pseudo-division, the Euclidean algorithm and its variants, Bézout's identity.
- Matrix algorithms: fast matrix multiplication, finding eigenvalues and eigenvectors, diagonalization and its application to matrix exponentiation, fast determinant calculation, division-free determinant algorithms.
- Square-free factorization of polynomials and its applications, symbolic integration of rational functions.
- Estimation and isolation of polynomial roots: upper and lower bounds for polynomial roots (e.g., Cauchy's bounds, Hong's bound, Laguerre–Samuelson theorem), Sturm's theorem, Descartes' rule of signs.
- Explicit formulas for the roots of low-degree polynomials, Abel–Ruffini theorem.
- Polynomial root approximation: bisection and Newton–Raphson methods, other approximation methods (e.g., regula falsi, secant, Laguerre's methods, QIR algorithm).
- The resultant of two polynomials, Sylvester's formula, connection to the Euclidean algorithm, the relationship between the resultant and the associated matrix, elimination and extension theorems.
- Discriminant of a univariate polynomial.
- Gröbner bases: monomial orders, S-polynomials and Buchberger's criterion, Buchberger's algorithm, minimal and reduced Gröbner bases, elimination using Gröbner bases.

4. Theory of probability

- Elements of combinatorics.
- Axioms of a probability space.
- Classical and geometric probability models.
- Conditional probability, the law of total probability, Bayes' formula.
- Independence of events and families of events, the Borel–Cantelli lemma, Kolmogorov's 0–1 law.
- Random variables, their distributions, cumulative distribution functions and probability densities, examples of discrete and continuous distributions.
- Numerical characteristics of random variables: expectation, variance, standard deviation, moments.
- Generating functions and their applications.
- Probabilistic inequalities: Markov's, Chebyshev–Bienaymé's, and Hölder's inequalities.
- Random vectors, multivariate distributions, marginal distributions, independence of random variables.
- Covariance, covariance matrix, correlation coefficient.
- Multivariate normal distribution.
- Characteristic function and its properties.
- Modes of convergence of random variables: almost sure convergence, convergence in probability, convergence in distribution, and the relationships between them.
- The relationship between convergence in distribution and pointwise convergence of characteristic functions.
- Limit theorems: the central limit theorem, laws of large numbers.
- Conditional expectation.
- Discrete-time martingales.

- Discrete-time Markov chains.

5. Differential Equations

- Existence and uniqueness of solutions of the Cauchy problem: Peano's, Picard's, Cauchy's theorems, Euler's method, method of successive approximations. Dependence on initial data and parameters, solutions defined on the maximal interval of existence. Cauchy-Kovalevskaya theorem for partial differential equations.
- Systems of linear first order ODEs: fundamental solutions, Liouville's formula, variation of constants formula, systems with constant coefficients.
- Linear higher order ODEs.
- Critical points of autonomous systems: Lyapunov stability, planar phase portraits, Grobman-Hartman theorem, Lyapunov stability criteria.
- Elliptic equations: Laplace's equation, fundamental solution, harmonic functions, maximum principles and uniqueness of solutions for the Dirichlet problem, Harnack's inequality, Poisson's equation, Green's representation formula, Poisson's formula for a ball, eigenvalues of symmetric elliptic operators.
- Parabolic equations: heat equation, fundamental solution, solution of the initial-value problem, maximum principle and uniqueness of solutions for the Dirichlet problem, separation of variables, Fourier transform.
- Hyperbolic equations: wave equation, d'Alembert's formula, Kirchhoff's formula.
- Sobolev spaces: weak derivatives, Poincaré's, Gagliardo-Nirenberg and Morrey's inequalities, Sobolev embeddings, Rellich-Kondrachov theorem, Rademacher's theorem.
- Weak solutions of second order elliptic equations: Lax-Milgram lemma.
- Basic notions of distribution theory.

6. Topology

- Methods of generating topologies.
- Closure and interior of a set. Dense sets, nowhere dense sets.
- Subspace. Induced topology.
- Separation axioms.
- Continuous mappings, homeomorphisms.
- Quotient spaces and quotient mappings.
- Metrization theorems.
- Paracompact spaces. Paracompactness of metrizable spaces.
- Cartesian product of topological spaces.
- Compactness of topological spaces, characterization of compactness in metric spaces.
- Topological characterization of the Cantor set.
- Tychonoff's theorem. The universal space for all Tychonoff spaces of weight κ.
- Cardinal invariants in topological spaces: the weight of a space, the character of a space, the character at a point, the Suslin number, the Lindelöf number, the density of a space.
- Complete metric spaces, Cantor's theorem.
- Čech complete spaces, Baire's category theorem.
- Connected spaces.