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Global central limit theorems for stationary Markov chains

Let $P = P(x, A)$ be a Markov transition probability on a general state space (S, Σ), with invariant probability m. Let $\Omega := S^{\mathbb{N}}$ be the space of trajectories with σ–algebra $\mathcal{A} := \Sigma^{\otimes\mathbb{N}}$, and let $\mathbb{P}_m$ be the probability on $\mathcal{A}$ of the chain with transition probability P and initial distribution m. By invariance of $m, \mathbb{P}_m$ is shift–invariant on $(\Omega, \mathcal{A})$ Let X_n be the projection of Ω on the nth coordinate. Then (X_n) on $(\Omega, \mathcal{A}, \mathbb{P}_m)$ is a stationary Markov chain with state space S.

We assume m ergodic for P, so the chain is ergodic too, i.e. the shift on Ω is ergodic.

We say that a real centered $f \in L_2(m)$ satisfies the *annealed* CLT if in $(\Omega, \mathbb{P}_m)$ we have

$$\frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma^2), \qquad \text{where } \mathcal{N}(0,0) := \delta_0.$$

We say that a real centered $0 \neq f \in L_2(m)$ satisfies the L_2–*normalized* CLT if

$$\frac{1}{\sigma_n(f)} \sum_{k=1}^{n} f(X_k) \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1),$$

where $\sigma_n(f) := \| \sum_{k=1}^{n} f(X_k) \|_2 > 0$ for large n.

We study conditions which yield that for every centered $0 \neq f \in L_2(m)$ a non–degenerate ($\sigma^2 > 0$) annealed CLT and an L_2–normalized CLT hold.

Joint work with Christophe Cuny