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GEOGEBRA AS A CONSTRUCTIVISM TEACHING TOOL FOR VISUALIZATION GEOMETRY USING VR AND AR

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Abstract: *GeoGebra software, with its constructivist teaching approach, proves to be a powerful tool for visualizing geometry and preparing future teachers for an immersive educational experience using Virtual Reality (VR) and Augmented Reality (AR). By integrating VR and AR capabilities into GeoGebra, educators can revolutionize geometry learning, offering students a dynamic and interactive environment to explore geometric concepts. With VR, students can step into virtual geometric worlds, enhancing their spatial understanding and promoting hands-on learning. They can manipulate shapes, explore 3D constructions, and visualize abstract concepts in a tangible way. On the other hand, AR complements real-world environments by overlaying virtual geometric elements, making learning contextual and engaging. For future teachers, this integration serves as a valuable training ground for incorporating cutting-edge technology into their classrooms. As they experience combined VR and AR-enhanced geometry lessons, they are gaining their confidence and expertise. In this way, GeoGebra empowers educators to create transformative learning opportunities, fosters their deeper understanding of geometry. It also nurtures a generation of tech-savvy teachers ready to shape the future of education. In this contri-*

bution, we present our experience with the use of GeoGebra software in combination with VR and AR in the geometry courses for future mathematics teachers. Results of qualitative research indicate the beneficial influence of GeoGebra software on the enhancement of students' geometrical competences during their teacher training.

Keywords: GeoGebra, Virtual Reality, Augmented Reality, teacher training

INTRODUCTION

In training future mathematics teachers, a primary task is to guarantee the level of their education in geometry. High school graduates come to universities with varying levels of knowledge in this area. During the geometry courses, we try to bring them to a higher level of understanding and remove any misconceptions they may have brought with them to college.

Van Hiele's (1986) levels of geometric thinking emphasize the gradual development of students' ability to understand and analyse geometric concepts. The first level, visualization, is a foundational step in this development and is critical for the further development of geometric thinking. Given the importance of visualization in geometric thinking, it is essential that geometry instruction begin with this basic step and gradually lead students to higher levels of abstract and critical thinking.

According to Duval (1999), visualization is not merely about seeing but involves cognitive processes to interpret and understand what is seen. Thus, the act of visualization in mathematics is a multi-faceted process that goes beyond mere imagery and involves various representations to facilitate understanding (Duval, 2006).

If visualization is considered as a key building block in geometry teaching, a crucial issue is the appropriate choice of visualization tools and measuring their impact on the effectiveness of teaching.

In the past few decades, digital technology has left an indelible mark on the educational sector, shifting paradigms, and introducing innovative methods to impart knowledge. One such transformative tool is the GeoGebra software, which offers a platform combining geometry, algebra, spreadsheets, graphing, and calculus (Hohenwarter & Preiner, 2007). Rooted in constructivist pedagogy, this software enables learners to construct their knowledge actively.

But as the technological landscape has evolved, so has the need to integrate more advanced and immersive technologies. The integration of Virtual Reality (VR) and Augmented Reality (AR) into GeoGebra heralds a new era in geometry education, leveraging immersive experiences to make abstract geometric concepts more tangible. At the same time, these approaches can be combined with more traditional ones to demonstrate alternative ways to problem solving. Such combination leads to deeper comprehension and allows discussing different faces of the problem (Lovászová & Hvorecký, 2002).

In our paper, we describe our experience with connecting the capabilities of the dynamic GeoGebra software with VR and AR environments in the classroom and seek to answer whether such a revolutionary form of teaching is a suitable means of visualizing geometry.

1. THEORETICAL BACKGROUND

1.1. The Constructivist Paradigm in GeoGebra

Constructivism posits that learners actively construct their own understanding and knowledge through experiences and interactions with the world (Piaget, 1952). Vygotsky & Cole (1978) suggest that knowledge is co-constructed through interpersonal interactions and cultural tools. Constructivist classrooms often emphasize problem-based learning, collaborative projects, and real-world applications to facilitate meaningful knowledge construction (Brooks & Brooks, 1993). In contrast to traditional educational approaches that might see learning as the passive reception of information, constructivism views learners as active agents, interpreting and creating knowledge based on their unique experiences and schemas (Fosnot, 1998). Thus, the learner leads learners to discover new ideas independently (Hejný & Kurina, 2001). GeoGebra, as a dynamic mathematics software, aligns well with these principles. Its dynamic interface facilitates active geometric exploration (Stahl, 2014). Users can manipulate objects, observe changes, and draw conclusions from these changes, actively constructing their understanding of geometric concepts.

GeoGebra, with its interactivity, offers a dynamic platform to engage students in geometric investigations, aiding in deepening their spatial understanding (Hohenwarter & Jones, 2007). GeoGebra supports multi-representational views, enabling learners to transition between 2D and 3D visualizations, thus strengthening their spatial visualization capabilities (Laborde, Kynigos, Hollebrands, & Sträßer, 2006). Research by Edwards & Jones (2006) showed that the integration of GeoGebra in geometry teaching improved students' problem-solving skills, as they could visually test and verify geometric properties and theorems. With GeoGebra, the abstract nature of many geometric concepts becomes more concrete, leading to increased student engagement and motivation (Arbain & Shukor, 2015). The software's dynamic interactivity provides an environment for students to explore, conjecture, and validate their understanding, promoting a deeper conceptual grasp (Dick & Hollebrands, 2011).

As educators increasingly leverage technology to foster spatial understanding, GeoGebra remains a vital tool in deepening students' geometric intuition and spatial reasoning abilities (Hohenwarter, Hohenwarter, & Lavicza, 2009).

1.2. Visualization: Bridging Abstract and Real-World Geometry

Visualization connects abstract geometric concepts with concrete real-world phenomena. Thus, it helps in translating complex relationships or properties into tangible experiences. As Zimmerman & Cunningham (1991) note, "Geometric representations and constructions serve as bridges between concrete manipulations and abstract reasoning".

Visualization in geometry helps bridge cognitive gaps by making implicit knowledge explicit. Arcavi (2003) observes that "often the visual representation can provide insights and understanding where other representations fail".

The need to coordinate among various representations (like symbolic, verbal, and graphical) is vital for comprehension. Visualization aids this coordination. Duval

(2006) emphasizes the necessity of transitioning between different semiotic representations in mathematical cognition.

According to Bishop (1980), visualization is not just for those with a propensity for it but can act as a mediator for all types of learners, enhancing understanding. It is thus suitable for diverse learning styles, making geometry more accessible.

Visualization provides context, linking theoretical knowledge to real-world applications. Giaquinto (2007) suggests that visual thinking in geometry can bridge the gap between abstract thought and tangible, real-world geometric situations.

1.3. VR and AR: Bridging Real and Virtual Geometry

Virtual Reality (VR) and Augmented Reality (AR) technologies have revolutionized the pedagogical landscape, particularly in the realm of geometry teaching, fostering enhanced spatial visualization (Sorby, 2009). Integrating these technologies into classrooms has shown potential to reduce the abstraction of geometric concepts, making it more learner-centric and interactive (Kaufmann & Schmalstieg, 2002).

Research by Kellman et al. (2022) highlighted that VR and AR applications in geometry can improve students' spatial reasoning capabilities, bridging the gap between 2D representations and 3D mental models. VR and AR are groundbreaking technologies that can significantly enhance the teaching of geometry by deepening students' spatial imagination. Such technologies connect theoretical knowledge and practical application, making geometry more accessible and relatable for all learners. The hands-on experience provided by VR and AR can help students develop a more intuitive understanding of geometric properties and spatial relationships.

Using VR technology, we can enrich students' mathematical modelling skills to address real-world issues (Cahyono, 2023). They will become more active in taking part in activities related to mathematics. Similarly, a mobile application with AR features can be a useful problem-solving tool to bridge the gap between real-world situations and mathematical concepts (Cahyono et al., 2022).

With AR, students can merge theoretical geometric constructs with real-world objects, grounding abstract concepts in tangible experiences (Ibáñez & Delgado-Kloos, 2018). AR overlays digital information onto the real world, offering students the chance to manipulate and study geometric objects in their immediate surroundings, making abstract concepts tangible.

2. METHODS

2.1. Survey

The study took place during the summer term of the 2022/2023 academic year and at the beginning of the winter semester of the academic year 2023/2024 at the University of Ostrava. It was held as part of a geometry courses designed for prospective mathematics teachers, with participation from 19 first- and second-year bachelor's degree students specializing in teaching for the second grade of elementary schools. The methodology of the research was grounded in qualitative analysis, employing action research with a focus on the method of observation.

In our observations, we focused on the improvement of students' geometric competences in relation to the use of GeoGebra applications, together with VR and AR technologies. According to National Council of Teachers of Mathematics (2000), geometric competence encompasses a range of skills and understanding in the field of geometry, crucial for navigating various academic and real-world contexts. It includes the ability to recognize and understand the properties and relationships of geometric shapes, as well as the skill to apply geometric concepts in problem-solving scenarios.

2.2. Training Future Teachers: Building Confidence and Expertise

Future mathematics teachers at the University of Ostrava take a course in planimetry and stereometry in the second and third semesters, where they could deepen their knowledge in the field of plane and spatial geometry.

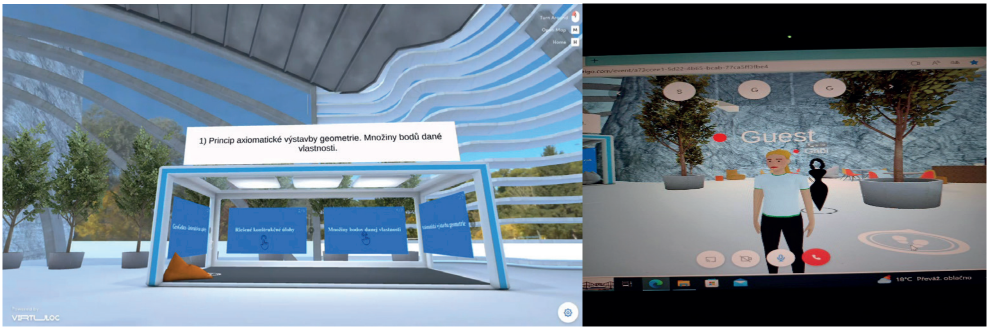
The course in planimetry begins by establishing basic geometric principles in the Euclidean plane. We build their theoretical geometric knowledge based on the principle of axiomatic construction of geometry. Students verify the validity of these rules by simple construction tasks by finding the intersection of sets of points of a given property. Gradually, we build into these problems the need to use more complicated concepts, such as congruent mappings. During the course we develop their classical drawing skills as well as their digital competence by using the GeoGebra software.

The course in stereometry follows planimetry. Here a fundamental problem appears. Students are not always able to grasp the causal relationships and connections with geometric concepts in the plane and in space. Most of them have negligible knowledge of spatial geometry, they know little about basic 3D shapes, and we can build their knowledge from scratch.

2.3. Implementation: Experiencing VR and AR in Teaching Geometry

Powerful information and communication technologies (ICTs) allow us to see virtual reality environment (VRE) as a tool to modernise the delivery of knowledge and skills, particularly to enhance the engaging nature of university humanities courses with interactive, practical elements (Korenova et al., 2022). According to Duhaney & Young (2011), the essence of this modernisation depends on effective communication between lecturers and students using modern ICT. VRE provides opportunities for students to engage and experiment. The sense of presence in this virtual environment is characterized by real-time interactions in 3D space and the representation of users through avatars, as reported Tachi (2013).

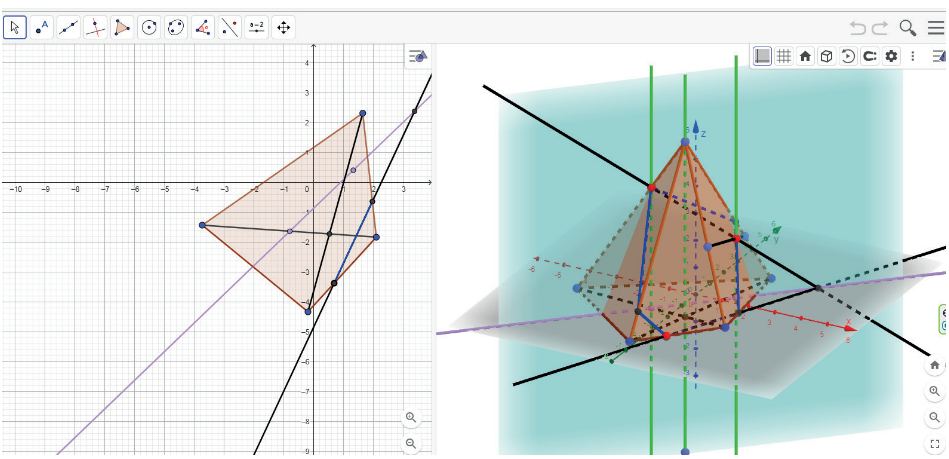
We have moved part of the classroom into the VRE, especially home training in the form of blended learning. GeoGebra applets implemented in this environment fulfilled the function of visualization and had a positive impact on the development of algorithmic thinking and spatial imagination of students (Schmid & Korenova, 2022). Students enter the VRE through avatars, which reduces the risk and fear of failure and thus increases their intrinsic motivation (Korenova, Hvorecky, & Schmid, 2023). They can interact with each other, creating a space for collaborative learning, that fostering teamwork, communication, and critical thinking skills , and communicative competencies (Tran, Pytlik, & Kostolányová, 2020).



F i g u r e 1. Blended learning in VRE
S o u r c e: Own work, 2023.

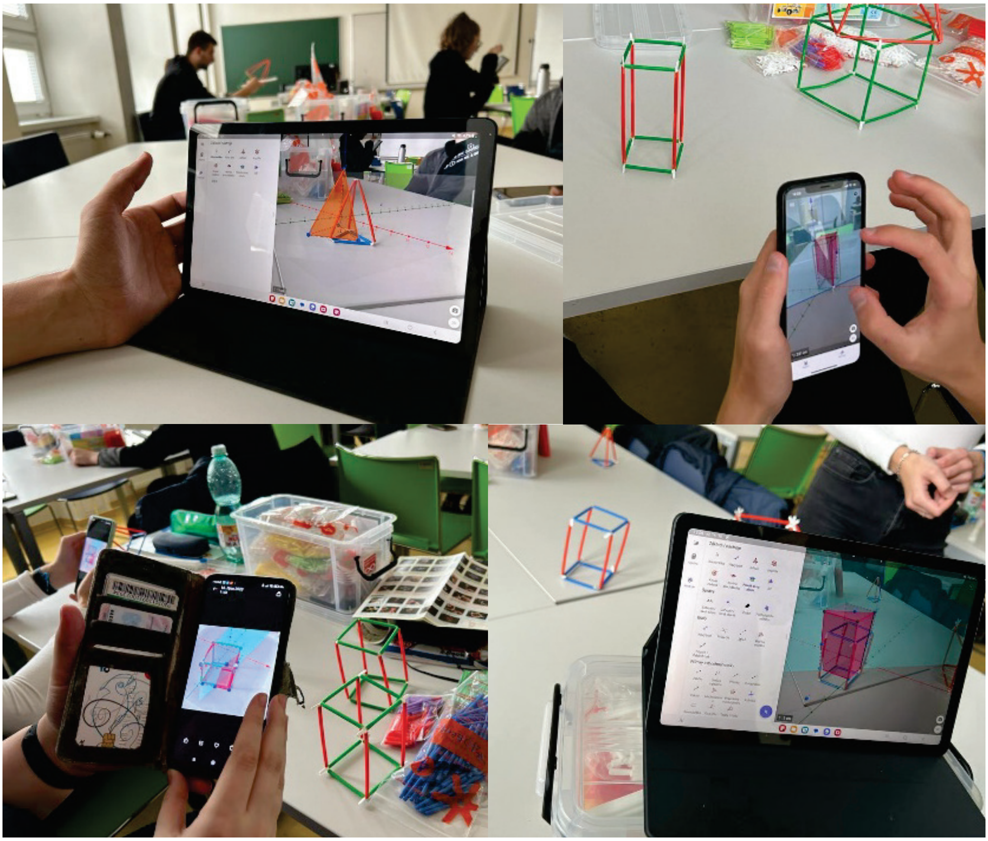
We also focused on this aspect: In selecting appropriate teaching materials, we should not overlook the importance of their attractiveness and ability to engage students if we are to achieve a deep and relevant understanding of the knowledge that these materials convey (Hvorecky & Korenova, 2023). Our innovative teaching model fully met this requirement.

We wrapped up our planimetry course with a gamified test in the VRE, aimed at assessing our students' capacity to apply their learned knowledge in problem-solving. During the stereometry course we teach the students positional problems, such as finding the intersection line of two planes or the intersection point of a line with a plane. They then apply this theoretical knowledge to solve practical problems, such as slicing solids through planes. This is where GeoGebra3D comes in as a great visualisation tool (Figure 2). The ability to move space allows us to see the situation from different angles. We can vary the position of given objects, which changes the lens of the object created by the construction. This contributes to a better spatial visualisation and subsequent understanding of the geometric context.



F i g u r e 2. Visualisation of knowledge used by slicing solids through planes
S o u r c e: Own work, 2023.

The AR application in GeoGebra also helps us to better visualise the situation when we embed the virtual model from the GeoGebra into real space. This can overlay a previously constructed real solid model, as shown in Figure 3. This is how we teach students to understand the relationship between the real model and its virtual representation.



F i g u r e 3. Overlaying real solid model used GeoGebra AR app

S o u r c e: Students' work, 2023.

To better understand the causality of relations and properties between planar and spatial elements, it is useful to transform the topic, the set of points of a given property, from plane to space. Adding another dimension complicated this task, but augmented reality allowed students to explore, verify and understand this topic. Moreover, they worked in groups during this activity (Figure 4), so that the knowledge they discovered could be discussed and argued with each other.

Figure 4. Modelling a set of points of a given property used GeoGebra AR app

Source: Students' work, 2022.

3. DISCUSSION AND CONCLUSION

To assess the level of our students' geometric competence, we used the results of the gamified test in the VRE, which consisted of 10 practical tasks. The average success rate of the group of 19 students was more than 83 percent.

In addition, we observed a cognitive increase in geometry comprehension in our students' use of the GeoGebra AR application. Based on this pilot study, we would like to investigate this interesting phenomenon in more detail with a larger sample of students in the future. Our main aim is to make learning valuable and attractive for our students. As Hvorecký (2007) said: "Mathematics Teaching will continue until people value it as a vivid, vital and no-nonsense subject".

Thus, in conclusion, we can confirm that GeoGebra software with the integration of VR and AR capabilities is a suitable tool for visualizing geometry as well as developing geometric and digital competencies.

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