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EXPLORING ADDITION AND SUBTRACTION STRATEGIES WITH VIRTUAL MANIPULATIVES ON TABLET DEVICES IN SECOND GRADE

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Abstract: *The purpose of this study was to observe how second-grade pupils with little previous digital experience explore various addition and subtraction strategies using several virtual manipulatives on tablet devices. During a one-week instruction pupils interacted with four virtual manipulatives and reacted differently to the each of them. Pupils' opinions, preferred addition and subtraction strategies and difficulties with the use and understanding of virtual manipulatives are described in this contribution.*

Keywords: virtual manipulatives, addition and subtraction strategies, tablets

INTRODUCTION

Since the introduction of virtual manipulatives many researchers have studied their potential or impact on student's learning (Baturo, Cooper & Thompson, 2003; Moyer, Niezgoda, & Stanley, 2005; Moyer-Packenham et al., 2014; Steen, Brooks, & Lyon, 2006; Suh, 2005). Tablet and mobile devices have brought new possibilities for their use and potential to engage students in mathematics and increase their motivation (Attard & Curry, 2012; Calder & Campbell, 2015; Goodwin, 2012; Hilton, 2018). Tablet devices are easy to use and children interact with tablet devices more freely and independently than with personal computers (Geist, 2012). It is necessary to explore its potential and study, how can these tools be effectively used to enhance students' learning and understanding of complex and abstract mathematics concepts.

1. LITERATURE REVIEW

1.1 Teaching with virtual manipulatives on tablet devices

Many mathematics apps contain a virtual manipulative that students can manipulate to support the visualization of mathematics concepts (Anderson-Pence et al., 2014). Students who struggle with abstract concepts can understand these concepts when given the opportunity to manipulate and explore mathematic objects in a problem solving environment (Bos, 2009). The use of iPads allows teachers to introduce and implement a wide range of teaching strategies (Attard & Curry, 2012; Spencer, 2013) and explore various strategies when solving problems. The multisensory interaction with tablet devices provides interactive manipulatives supporting transition between concrete and visual representations and abstract knowledge (Volk, Cotič, Zajc & Starčič, 2017).

A recent study conducted with ten students of primary pedagogy tutoring 10 lowachieving primary school pupils revealed that supplemental teaching with iPads can help pupils understand numeracy concepts and improve their math skills (Kaur, Koval, & Chaney, 2017). Similarly, Spencer (2013) reported that the usage of iPad can improve students' numeracy learning. The results from a research done in two Californian primary classrooms showed that iPads can support the development of concepts, critical thinking and pupils' learning and achievement (McKenna, 2012).

There are many advantages of tablet learning. Culén and Gasparini (2011) reported that the use of iPads supported collaboration, interaction and discussion among pupils. Children work on tablets intuitively in groups and want to share their experience with their peers (Shifflet, Toledo, & Mattoon, 2012). Discussion and collaboration along with the use of multiple representations and various teaching strategies support pupils' development of concepts and construction of solid mental representations. To understand the concepts and relations through manipulatives, it is necessary to use them effectively, so that they would really represent the concept and help the students. Greeno and Hall (1997) argue that pupils need to use multiple representations in various situations while communicating and reasoning in the context of a social environment.

1.2 Addition and subtraction algorithms

When teaching addition and subtraction strategies up to 100, teachers can choose from several strategies and demonstrate these with multiple enactive, iconic or symbolic representations. When pupils use appropriate models, they are able to discover when and how to use the concepts of composing, decomposing, regrouping or 'make ten' strategy. Manipulatives displaying the principles of addition and subtraction in our number system are e.g. the base ten block, number rack and number frames. Additional strategies can be discovered also by using the number line. These are now accessible as online learning tools or apps available for tablet devices.

One of the virtual manipulatives, the virtual base ten block, shows pupils the principle of regrouping tens and one when adding or subtracting. In a virtual manipulative, pupils don't need to replace one rod with ten units. They can tap on the rod to change it into ten units or circle ten units to connect them into one rod.

Number rack and number frames are used in similar manner. Double-coloured counters placed on a number rack demonstrate concepts of composing and decomposing and at the same time, pupils see the two addends in an addition problem or the minuend and subtrahend in a subtraction problem. When modelling the numbers with double-coloured counters, pupils can see, which of the two numbers they need to decompose and they can change the colour of counters when needed.

A different strategy for addition and subtraction needs to be implemented on a number line. Its way of use represents mental addition and subtraction. At first, pupils need to add or subtract tens, and then the ones by jumping from the starting number. Rather than jumping 36-times, pupils can jump three-times ten and then jump six-times by one space.

These and other strategies can be useful for pupils when learning addition and subtraction algorithms. When a teacher introduces more strategies, pupils get the opportunity to choose the most suitable strategy according to their abilities and get more chances to understand the concepts and relations through representations that can be meaningful to them. There are many virtual manipulatives at teacher's disposal, however, it is necessary to use them according to the pupils' abilities or their learning styles.

This study aimed to explore, how pupils using mostly symbolic representations interpret and react to virtual manipulatives. The purpose of the study was to observe, how pupils with lack of digital experience explore addition and subtraction strategies with virtual manipulatives on tablet devices. There were two research questions:

- How will pupils interact with different virtual manipulatives on tablet devices?
- What addition and subtraction strategies will they use?
- Which advantages and disadvantages will they discover?

2. METHODS

Research design is based on qualitative approach consisting of intervention and direct and indirect observation. 24 second-grade pupils participated in a one-week instruction during regular mathematics classes at the end of the school-year and used virtual manipulatives on tablet devices while exploring different addition and subtraction strategies through different virtual manipulatives. The instruction took place on the second and third lessons to prevent tiredness and lack of concentration. The lessons were instructed by a research member while the class teacher was observing the lessons. The class teacher received a printed structured observation sheet for this purpose. Pupils were asked to use tablet devices in pairs to encourage cooperation and discussion between them. We wanted to observe how pupils would explain the problems or manipulation use to each other. The class received an introduction for every virtual manipulative from a research member who explained how to work with the virtual manipulative frontally on the interactive whiteboard. Pupils were showed several examples and solved some tasks frontally before they used the virtual manipulatives in pairs. After each lesson pupils were asked to express their opinions concerning the use of the virtual manipulatives, explain whether or why they found the manipulative to be useful and state any difficulties they had encountered.

All lessons were video-recorded by two research team members focusing on pupils’ activities, way of working with the manipulatives, pupils’ discussions and ways of solving tasks on tablet devices. After the end of the instruction the class teacher was interviewed individually. The interview was audio-recorded and transcribed. Additional data was collected from class teacher’s notes recorded during the instruction. A structured observational sheet has been prepared for the class teacher. The qualitative data was analysed and coded using open and selective procedures into categories to identify emerging themes which were used for interpreting the data.

2.1 Participants and setting

The study was conducted in a second grade class in a small-town school with a lack of digital equipment. The school’s population is approximately 250 pupils and the only digital equipment accessible to the pupils are a computer classroom and a class with an interactive whiteboard. None of these are used by primary pupils. The instruction took place in the only classroom with the interactive whiteboard.

At the time of the study the class teacher was in her second year of teaching and preferred teaching through iconic and symbolic representations using mostly textbooks and workbooks. 24 pupils (13 boys and 11 girls) participated in the instruction. 20 pupils out of 24 had experience with tablet-based instruction from previous research they participated in the first grade, but they had not used any digital equipment at school on a regular basis. Only a few of them had access to tablet or mobile devices at home, but most of them had difficulty handling the device. The parents of the pupils have been informed about the research and have signed an informed consent before the beginning of the instruction. All parents have agreed for their children to participate in the study.

2.2 Instruments and data sources

During the instruction pupils used tablet devices with Android based operating system and the instructor used a Windows-based laptop connected to a Smart board. All tablets and the laptop were connected to the Internet through the school's wireless network and in case of any difficulties with the connectivity, a few extra tablets were prepared. We have chosen to use online virtual manipulatives which were compatible with both the Windows-based laptop used by the instructor and the Android-based tablet devices. We decided to use the free math apps available through The Math Learning Centre's resources (https://www.mathlearningcenter.org/resources/apps). However, we could use only the web apps, but couldn't download the apps to the tablets due to compatibility problems.

Pupils used four virtual manipulatives available on the website:

- Number Frames.
- Number Line.
- Number Pieces.
- Number Rack.

Two research team members recorded all lessons with two digital cameras from different perspectives. Additional materials include the structures observation sheets prepared for the class teacher, the structured interview guide prepared for the interview with the class teacher and the lesson plans prepared by the instructor.

3. RESULTS

When pupils received the tablets, they showed increased motivation and excitement and verbally expressed their happiness of tablet usage. The instructor demonstrated the basics of tablet usage, but most of the pupils remembered its use from the previous research. During the instruction they had a few difficulties handling the device, such as accidentally closing the browser or problems with connectivity, but they did not have any significant difficulties. On each lesson they explored other virtual manipulatives. The manipulatives were explored in the following order: number frame, number rack, base ten block and number line. The number frame and number rack were used during the first day of instruction, the base ten block on the second day and the number line on the third day. The four manipulatives will be analysed separately.

3.1 Virtual number frame

Pupils had little previous experience with number frames and double - coloured counters in form of physical manipulatives from previous research. They had not used them on regular math lessons with their class teacher. In this research, Number Frame (https://www.mathlearningcenter.org/resources/apps/number-frames) virtual manipulative from The Math Learning Centre website was used. Pupils worked only with the 100-frame. Pupils needed little instruction on how to use the counters, but had difficulties placing a set of ten counters on the right place in the frame (see Figure 1). One of the first advantages discovered by pupils was the possibility of moving more counters at the same time. They declared that this feature of the virtual manipulative made the use easier and faster. Pupils were asked to solve several addition and subtraction problems.

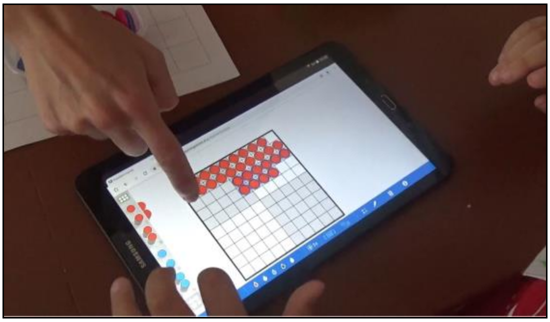

Figure 1. Placing a set of ten counters

Source: Screenshot from the video-recording. Own work

When adding a one-digit number to a two-digit number, some pupils solved tasks mentally. They did not need to use the manipulatives, because they could already solve these tasks. Approximately half of the pupils relied on the virtual manipulative and a few of them counted the counters and checked the results several times by each task (see Figure 2).

When adding two two-digit numbers, all pupils placed the counters correctly. However, many of the pupils wanted to solve the tasks mentally instead of telling the answer from the number frame, and solved the tasks incorrectly. At first, pupils had difficulties with decomposing the second addend in spite of having the counters placed correctly in the frame. The instructor decided to solve some tasks frontally on the interactive whiteboard and helped student with her questions to analyse the relations. After a few examples, pupils could explain the principles and how to decompose the second addend.

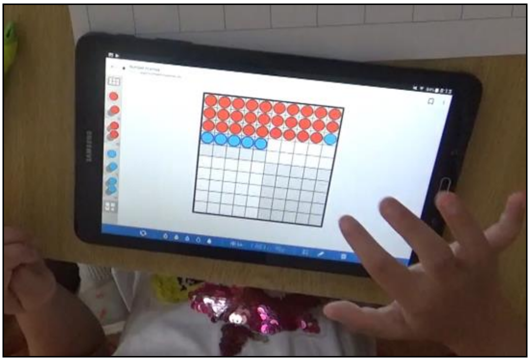

Figure 2. Adding on the number frame
Source: Screenshot from the video-recording. Own work

This virtual number frame was relatively simple for the participants. They understood its use and the order of the counters helped them understand an algorithm of mental addition. Pupils stated that this virtual manipulative was the most useful for their learning and explained that the order of counters (ten counters in ten rows) resembled the order of beads on a number rack.

3.2 Virtual number rack

Pupils were familiar with the use of the number rack from the first grade, although they had previously used only physical number racks with twenty beads. The only help needed was the way to display 10 rows of beads. Pupils found the virtual manipulative's use easier than the use of a physical number rack. They did not need any additional instruction and solved the addition and subtraction tasks easily. Pupils were immediately able to explain the number rack's use and explain the tasks using the virtual manipulative. They explained that it is easy to add tens on a number rack and were able to explain the algorithm which is used for mental addition.

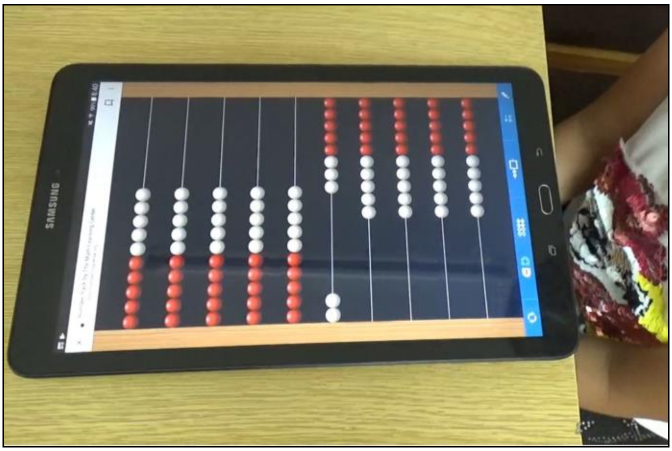

Figure 3. Displaying the result on the number rack
Source: Screenshot from the video-recording. Own work

Pupils did not mention the fact that they could not see the addends or the minuend and subtrahend at the same time with the result on the number rack (see Figure 3). They displayed the first number, added or subtracted the given number and were confident that they solved the task correctly. When solving addition problems, pupils used two strategies. Majority of pupils added the second addend by ones, adding the beads one by one. Some pupils counted the remaining beads in the row, added these beads as a set and then added the remaining beads from the next row explaining the principle of decomposing the second addend into two addends (see Figure 4). However, majority of pupils did not decompose the second addend, nor did they discover any strategies related to number five.

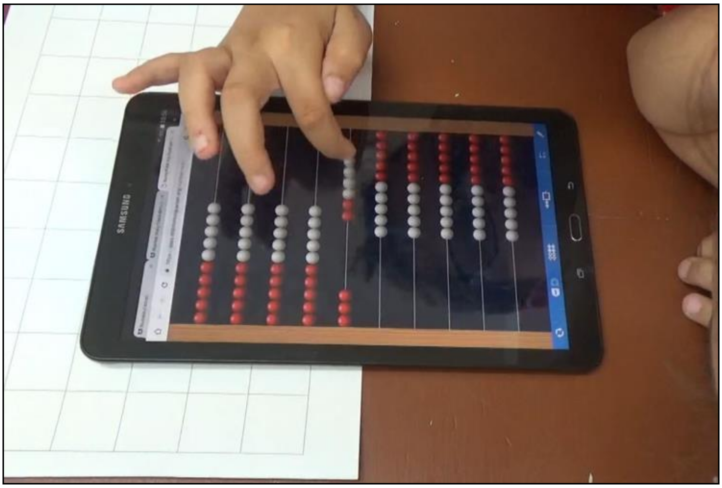

Figure 4. Decomposing the second addend

Source: Screenshot from the video-recording. Own work

3.3 Virtual base ten block

The virtual manipulative the Number Pieces (https://www.mathlearningcenter.org/resources/apps/number-rack) appeared to be complicated for the participants. They had problems with manipulating and organizing the rods and units and difficulties with counting the units. They often organised nine or eleven units into one rod with the same length as rods containing ten units not seeing the difference. Most pupils manipulated the rods and units in disorganised ways. Therefore, the most common strategy for solving the problems was counting all rods and units one by one. Pupils continued to use this strategy even after discovering that they can connect ten units into one rod by circling the right amount of units. Several pupils expressed in their opinions that working with this manipulative required too much time.

3.4 Virtual number line

The virtual number line was the last virtual manipulative used on the instruction. All pupils were familiar with the use of number line, but the only strategy they knew was moving by ones on the line (see Figure 5). After solving a few tasks frontally on the interactive whiteboard, they discovered that they could move also by tens (see Figure 6). They could explain and solve the tasks moving tens and ones frontally on the interactive whiteboard, but the majority of pupils jumped by ones when adding or subtracting two-digit numbers on the tablet devices. A few of them stated that they did not need to calculate, they could just jump on the line and write the answer. There were pupils who did not want to use the number line. They solved the tasks mentally and wrote the results.

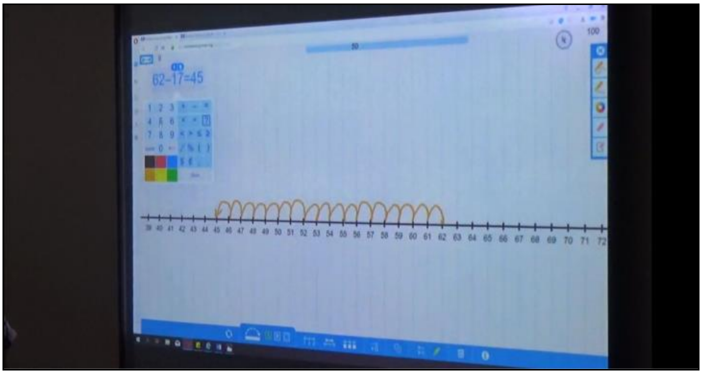

Figure 5. Moving by ones on the number line
Source: Screenshot from the video-recording. Own work

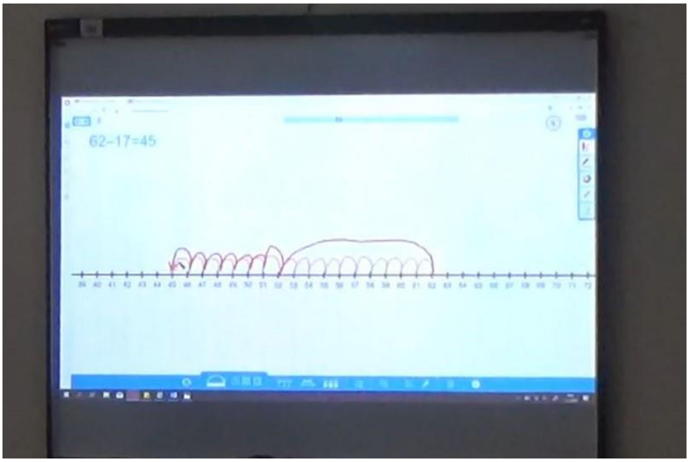

Figure 6. Moving by tens and ones on the number line
Source: Screenshot from the video-recording. Own work

CONCLUSION

During the instruction pupils were excited to use tablet devices. They talked about the activities as play and explored the possibilities of virtual manipulatives, mostly focusing on changing colour or randomly adding or deleting objects. Pupils were engaged with the tablet devices, but not always with the tasks. They explored the four virtual manipulatives on four lessons during three schooldays. It is possible that pupils' reactions and motivation were different during different days of instruction depending on their tiredness and mental state.

At first, pupils tried to solve the tasks mentally or using numbers. They needed time to adept to visual representations and some students failed to connect the visual and symbolic representations. They solved the task mentally and explained the algorithm with numbers, but could not relate the symbolic representation to the visual one displayed through the virtual manipulative. On the other hand, there were pupils who found the manipulatives useful and could explain how they worked.

There are several other studies reporting the same difficulties. A study conducted with third- and fourth-graders using virtual manipulatives revealed that many pupils had difficulties to interpret visual representations in mathematic tasks (Anderson-Pence et al., 2014). Westenskow and her colleagues (2014) found that third- and fourth-graders had difficulties especially with selecting the correct visual models and answering the problem goal. On the other hand, a study conducted with children regularly using physical manipulatives found that virtual manipulatives provided an important bridge between concrete, iconic and symbolic representations (Moyer, Niezgoda, & Stanley, 2005). They also reported that pupils preferred different methods for solving addition problems and changed their strategies when working with the virtual manipulatives for a few days.

Pupils in this study also discovered several strategies using the virtual manipulatives, but most pupils preferred to use only one or two strategies. Some pupils refused to use the number line and majority of pupils did not understand the use of the virtual base ten block. On the other hand, one girl preferred the use of virtual base ten block and had difficulties with using other virtual manipulatives.

The findings also show a difference in pupils' independence. Some pupils needed guidance during all lessons and had difficulties when working in pairs. They wanted more examples and did not want to use the manipulatives individually. Instead of using the manipulatives, they tried to solve the tasks mentally or using numbers, as they were used to. Similarly, a study comparing physical and virtual manipulatives revealed that pupils insisted on using the usual methods of problem solving refusing to experiment with new mathematics tools (Thompson, 1992). On the contrary, Highfield and Mulligan (2007) reported that pupils using virtual manipulatives explored and experimented more than pupils in the control group.

This study examined the reactions of pupils with lack of digital experience on tablet learning through virtual manipulatives. The study revealed that some pupils were able to understand the usage of virtual manipulatives immediately, while others had difficulties with connecting visual and symbolic representation. The selected virtual manipulatives supported the learning of mathematics concepts differently and were useful to different degrees according to pupils' individual needs. The use of virtual manipulatives on tablet devices can contribute to pupils' understanding and learning of addition and subtraction algorithms, but it might not be beneficial for all students. It supports differentiation, and pupils need more time to adapt to the device, learn the use of manipulatives and select the most suitable manipulative corresponding with their mental structures and thinking.

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