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DEVELOPMENT OF GEOMETRICAL THINKING VIA EDUCATIONAL SOFTWARE BY PUPILS OF ELEMENTARY SCHOOL

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Abstract: *The study is aimed to describe the using of geometrical educational software oriented to develop geometrical spatial thinking. This software is available on the webpage www.delmat.info. We would like to show his functions, propose concrete thematic areas in Slovak and Polish curriculum in the elementary level useful for this software. Future research in Polish elementary school in the 1-3 grades will be discussed.*

Keywords: Polish and Slovak math curriculum in elementary level, geometrical and spatial thinking, educational experiment during the mathematical lesson.

INTRODUCTION

According to Karolčík, Čipková, Veselský, Hrubišková (2013) educational applets belong to the smallest program units, which can be launched separately or as plugins of web pages. Their didactic usage is limited mainly to the mathematical calculations in more complex tasks, addition of missing data, explanation of problems or simulation of phenomena or processes. The interactive presentation mainly consists of animations supplemented with graphical and sound effects. Their progress can be influenced, e.g. by the change of the input quantities.

The mentioned study shows that prepared educational applets can be evaluated by three groups of respondents: information technology (IT) specialists, users - pupils or students and teachers.

IT specialists observe mostly, if the applets:

- are clear, simply and user-friendly;
- have good graphical processing and design variety;
- give the possibility of accessing the software through the internet;
- support clarity with illustrative examples, animations, pictures;
- have good interactivity.

Teachers also believe that the applets must be clear, interactive, simply and user-friendly and also, they must have good graphical processing and design variety. It is important for them that the applets:

- have completeness of content with multimedia elements;
- have comprehensibility and language mutation in mother tongue;
- Users-pupils and students has specific expectations, that the applets are;
- proficiency prepared and they bring perfection of data and information;
- suitable for students and pupils considering their age.

It is important in our case that geometry applets respect the abilities of pupils at elementary schools. We use these applets as a supporting tool for pupils' development of spatial thinking.

In Poland, the evaluation of teaching games is used mainly for formative evaluation (allowing to shape the scope of work on the didactic game) it is carried out before and during program development in order to optimize it.

It involves collecting and analyzing data. A distinction is made between two types of formative evaluation: process evaluation, which aims to evaluate the educational process, and performance (product) evaluation, which aims to evaluate the outcome of the educational process e.g., changing the level of spatial thinking of young students relative to the evaluation of other students who did not use computer program (Griffin, Butler, 2005).

Program or computer game evaluation includes two assessments: process evaluation and product evaluation. The first contains:

- interviews, surveys for teachers, parents or students, used e.g. to find out the attractiveness and level of acceptance of the program;
- observations regarding the educational process. They are especially popular at the initial stages of developing an information program;

- behaviour analysis can be applied using modular programs and in which a variety of teaching methods are used, not only computer-based.

Product rating includes:

- tests checking the knowledge of the subject;
- observations of behaviour in control and experimental groups. (Margulis, 2005).

The main goal of product evaluation is to determine which factors in the educational process determine the possible effects of the program. Detailed information obtained as a result of product evaluation should be the basis for program optimization.

Space thinking has an important role in the development of mathematical and logical thinking and it is also important for development of the competence of solving real-word problems. It is a task for teachers to prepare suitable educational activities for pupils in the frame of development of their space thinking. According to Sinclair, Bruce (2015) some new trends in geometry education at primary level are:

- the role of spatial reasoning and its connection to school mathematics in general and school geometry in particular;
- the function of drawing in the construction of geometric meaning;
- the affordances of digital technologies in geometric and spatial reasoning;
- the importance of transformational geometry in the curriculum (including symmetry as well as the isometries);
- extending primary school geometry from its typical passive emphasis on vocabulary (naming and sorting shapes by properties) to a more active meaning-making orientation to geometry (including composing/decomposing, classifying, mapping and orienting, comparing and mentally manipulating two- and three-dimensional figures).

According to Wai, Lubinski, Benbow (2009) space thinking in education supports STEM (science, technology, engineering and math) disciplines. They mean, that spatial ability has emerged as a salient psychological characteristic among pupils who go on to develop expertise in STEM domains.

Many pupils have problems with using spatial imagination in the solving of concrete tasks and they have a problem, for example, with building the solid, if they know the top, front and right view. We developed applets as a supporting educational tool, which help and supplement pupils' manipulative activities with solids.

1. THE ROLE OF GEOMETRICAL THINKING IN PRIMARY MATHEMATICS EDUCATION

Solving many real life problems involves geometrical thinking, which is based on visual concepts of geometric notions. We have carried out some research on pupils' understanding of geometric notions at primary level (Gunčaga, Žilková, 2019). This study shows, that many pupils have a problem with connecting the name of the plane geometric shapes with the properties of the shape. Another problem is to identify the geometric shape on the basis of given models (Gunčaga, Tkačik, Žilková, 2017). These problems are connected with the teacher training at universities. Many pre-service teachers can identify planar shapes only in standard position and they do not have an accurate understanding of the properties of the shapes or they are uncertain in terminology (see Žilková, Gunčaga, Kopáčová, 2015).

According to Hejný et al. (2006) the ability to manipulate with three-dimensional (3D) objects is built gradually. The starting point is pupils' direct tactile and visual experience with models of the mentioned objects. The gradual shift to mental operations goes through many stages in which each new one lacks a certain type of perception with regard to the previous one.

Jirotková (2010) states that these activities are focused on the idea that the pupil

- obtain enough experiences about object,
- know the object during the activity with it,
- can discuss about object with other pupils in his class or group,
- tries to define with own words the properties of the object,
- can change with the teacher's help his own definition of the object up to the correct definition.

Solids or 3D objects belong to an important part of geometric notions in primary education. According to Weigand (2009) the teaching of geometric notion has three important aspects: the building of reasonable imagination, acquisition of knowledge and acquiring skills.

If we built reasonable imagination about a geometric notion, then we perform with pupils some manipulative activities with concrete objects, perception at pictures and models, verbalization of geometric objects. Acquisition of knowledge is based on knowledge of the properties of geometric notions, relationships between mentioned properties and relationships between discovered a new geometric notion to the other already known notions. Acquiring skills by pupils is realized via the ability to work and manipulate with geometric objects, making simple constructions or calculations and also solving problems.

The development of information and communication technologies (ICT) brings possibility of supporting this process of pupils' understanding of geometric notions and spatial imaginations via the development of special educational geometric applets. We would like to present in our article some possible educational interventions in geometry education with the help of the mentioned applets.

2. SPATIAL THINKING ACCORDING TO THE NATIONAL CURRICULUM IN MATHEMATICS ON THE PRIMARY LEVEL IN SLOVAKIA AND POLAND

The Innovated Slovak State Educational Program Mathematics – Primary Education (ISEP(2014)) obtains in the first year in the thematic area "Geometry and measurement" the notion of spatial geometric figures: cube, cylinder, sphere. It is expected that the pupil can name the space geometric figures. He should be also able to draw and put these figures according to the teacher's instructions, to identify the place of geometric figures in the space.

The thematic area "Geometry and measurement" includes in the second year the topic ”Creating a building from cubes“. This topic assumes that pupils can build a simple building from cubes according to a template or a figure.

The solid cube is explained in the third year – vertexes, edges, faces. For this reason, it is expected, that children can build buildings from cubes according to a given plan – the top, right and front view. These plans obtain a marked number of stacked cubes. Children can identify rows, columns by building from cubes and also, they can create a plan – the top, right and front view with a marked number of stacked cubes according to a concrete building from cubes.

There is an extension in the fourth year – to create different buildings from cubes according to a plan – the top, right and front view, to create and verbally describe your own building from cubes and to draw a plan – the top, right and front view from a concrete building from cubes.

There exist differences between countries in the State Educational Curriculum, which will be described in the following part of the Polish Curriculum for development of spatial thinking on the primary level. The child begins his development of spatial thinking from mastering the orientation in the body schema and the direction-spatial orientation towards the immediate environment (Black, Walker, Fernald 2017).

There are the following spatial thinking requirements in primary education that fall within the scope of mathematical education: achievements in the field of understanding spatial relations and size characteristics.

The problem with spatial thinking in Polish education begins to appear in the mathematics lesson in the geometry classroom, it quickly becomes apparent which student has trouble understanding the topics, spatial vision. It turns out that this is a large group of 30% (Makarewicz, 1999). Problems start by lower elementary school pupils when they draw parallel and perpendicular lines. Pupils do not know how to operate a ruler and a set square, the arrangement of these two instruments arouses their emotions, they are often discouraged. Some pupils are very clever, but there is a group of pupils who, despite having geometric accessories, draw crookedly, do not see right angles, and have difficulties drawing precisely. It is difficult for them to see geometric dependencies, they do not understand the instructions in the textbook, and they are unable to plan the next stages of creating drawings or solving a geometric task (Makarewicz 1999).

In recent studies, Kopczyński, Gałuszka (2019) show on the example of two experimental and control groups how to increase the level of spatial thinking. For this purpose, the researchers used an educational mat that allows manipulation of objects on the matrix. Groups of pupils in primary education were tested for: solving mathematical problems, performing simple equations and geometric tasks that require advanced spatial thinking.

In the area of geometry skills, the control group achieved scores in a range of 0–6 points in the initial measurement (test K3 M1), averaging 2.20 points (SD = 1.65), usually 1 point. In the final measurement, the control group scored 0–6 points, averaging 2.74 points (SD = 1.88), usually 2 points.

In the area of geometry skills, in the initial measurement (test K3 M1), the experimental group achieved scores in the range of 0–6 points, averaging 2.46 points (SD = 1.67) out of 6 points possible, usually 2 points. In the final measurement, the group scored 0–6 points, averaging 3.55 points (SD = 1.80), usually 3 points (Kopczyński, Gałuszka 2019).

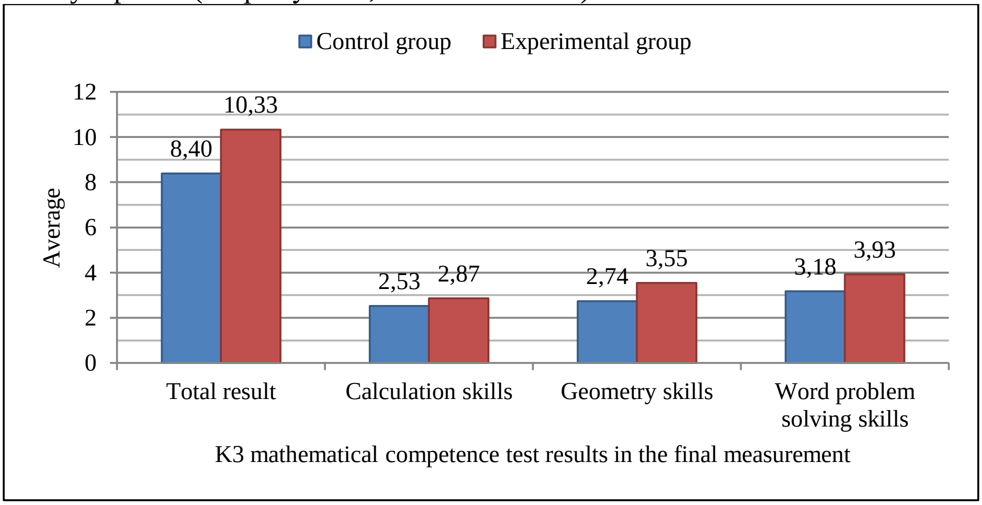

Figure 1. Comparison between the K3 M2 mathematical competence test results of the control and experimental group in the final measurement.
Source: Own work

The presented results show a significant statistical significance of the EduMata didactic aid (cubes and forms) on the results achieved in mathematical thinking among 3rd grade primary school children. There was a significant increase in general mathematical competences and a significant increase in calculation skills, geometry skills and word problem solving skills in the final measurement. Thus, the measurements show that the largest point difference between the groups in the final measurements noticed in the field of geometrical competence. This means that the manipulation of various objects on this stage significantly contributes to the development of spatial thinking.

3. SPATIAL THINKING IN THE CONNECTION WITH NATIONAL CURRICULUM IN COMPUTER SCIENCE ON THE PRIMARY LEVEL IN POLAND

Spatial thinking and spatial orientation in mathematics lessons along with the development of new technologies is directly related to the IT competences and information literacy of students. The development of skills such as creativity, innovation, critical and logical thinking skills of students can be shaped through tools and software that develop students' multifaceted thinking (for example, Delmat Applets). For a better understanding of spatial thinking, it is necessary to be able to use virtual, interactive orientation, and thus to support traditional teaching methods with modern tools. In addition, taking into account the specificity of the digital society in which modern children are brought up, an argument arises to use the natural potential of students to operate computer devices and programs for mathematical education. This way IT competences are transferred from the level of fun or free time activities to the field of school work methodology. "In this way, you could show students the relationship between mathematics and computer science. They could expand their knowledge by playing, learning about computer programs used for mathematical calculations and finally stimulate their own creativity, the ability to think logically and spatial imagination" (Maj, Falkiewicz 2018).

Using of Information and Communication Technologies (ICT) can help in the use of different kinds of models in mathematics education. These models can help to develop pupils' spatial thinking. For this reason, great importance in education has been attached to the use of computer applications, online resources and communication by covering all pupils with ICT education. Today's expectations of citizens' digital competences go beyond traditional computer literacy and technological proficiency. These skills are still needed; however, they are no longer sufficient at a time when computing is becoming a common language in almost every field, each of which is constantly being equipped with new tools. "Computational thinking that teachers develop among their students is a gain for students, and examples of programs that they propose to do will also have a triple benefit for students. They will strengthen the learning of content from the subject, students will create things important for themselves, the teacher will easily introduce to his arsenal of teaching resources and tools afforded by modern technologies. It will be true learning through creation, and yet it is the highest strategy for acquiring knowledge and skills (Rostkowska 2017).

Applets support the work of teachers and pupils with the interactive whiteboard. It is important, that the pupil can use notions top, right and front view. They have also propaedeutic function. Applets develop not only spatial imagination, but also informatics competences. If we draw spatial geometrical figures through planar figure on paper, then it is important that this figure evoke some spatial geometric figure.

The methods and techniques learnt in ICT classes should give pupils the skills of logical and algorithmic thinking, programming, using computer applications, searching and using information from various sources, using computers and basic digital devices as well as applying these skills for various purposes, such as making calculations, processing information and its presentation in various forms (including visual, see Rozporządzenie [Regulation], 2017a and b). Therefore, it can be concluded that the elements are necessary for computational thinking which is understood as "thinking that accompanies the processes of solving problems by means of computers, and which is characterized by the following features:

- The problem is formulated in a form that makes it possible to use a computer or other devices to solve it;
- The problem consists in the logical organization of data and its analysis, which can be facilitated by, among others, data models and model simulations;
- The solution to the problem can be obtained by using an algorithmic approach and therefore takes the form of a series of steps;
- The design, analysis and computer implementation of possible solutions leads to the most effective solution and to utilization of a computer's capabilities and resources;
- The experience gained in solving one problem can be used to solve other problem situations." (Sysło 2019)

4. DESCRIPTION OF VIRTUAL MANIPULATIVES WITH CUBES

One of the aims of the project APVV-15-0378 and the project KEGA 003TTU-4/2018 is to create learning environments for pupils of primary education such that they support their spatial imagination and orientation in the space. At the same time, all suggested educational environments and interventions should be projected so that they reflect abilities, interests and needs of children in young-school age. From this reason we created so called “mirror activities” with buildings from cubes. Under the term “mirror activities” we understand activities with the same contentual mathematical basis but realized in other educational environments. In this part of the paper we present justification for the design and description of educational mobile applications which provide interactive environment for creation of buildings from cubes, their record or interpretation, respectively. In doing so we assume that pupils will realize the same tasks using real cubes, grid paper and pencil at first. Interactive applets should create alternative educational environment; an environment for exploration, creation and verification of hypotheses, respectively. The task of educational applets is to help pupils in the transformation of spatial and planar representations of buildings from cubes.

In the process of creation of educational applets focused on virtual manipulation with cubes we came from the paradigm of research method Design Based Research (DBR). DBR assumes cyclical repetition of educational interventions, their evaluation and adjustment of intervention according to observed research findings from pedagogical praxis. The result of multiple repetition of development of applets is 5 educational interactive products designed for pupils of primary education:

Applet No 1. The goal of the applet available at http://www.delmat.info/a/8b/ is to develop an ability of pupils to **create a building from cubes according to the plan.**

Applet No 2. The goal of the applet available at http://www.delmat.info/a/8d/ is to develop an ability to **create a plan according to a building from cubes.**

Applet No 3. The goal of the applet available at http://www.delmat.info/a/8c/ is to develop an ability to **create three views (top, front and right view) of a building from cubes.**

Applet No 4. The goal of the applet available at http://www.delmat.info/a/8a/ is to develop an ability to **create a building from cubes according to three views.**

Applet No 5. The goal of the applet available at http://www.delmat.info/a/8e/ is to develop an ability of critical thinking and so to f**ind a mistake in a building from cubes and correct it** so that it corresponds to the three views.

Applets No. 1 to No. 5 are ordered according to the level of complexity for pupils. Research within the DBR methodology showed that creating a building from cubes according to the plan is for pupils an easier task than to create the plan of a building. Similarly, working with three views is more difficult in terms of spatial imagination than working with a plan. Pupils considered the task about finding and correcting mistake (applet No. 5) the most difficult. Therefore, we recommended to include this task to the highest grades of primary education and going beyond to the low secondary education.

Applets provide so called "inverse tasks" which means that a pupil should be able to create a building (create a real or virtual model) according to a planar representation (plan or three views) but also an inverse task – to create a planar representation of a building from cubes on the basis of a virtual interactive model of a building from cubes. Applets No. 1 and No. 2 generate inverse tasks with a focus on the ability to understand and know how to interpret a plan of the building (Figure 2, Figure 3). A plan of a building is a unique representation of model of building, therefore, there was not any problem with the implementation of the applet from the point of view of the evaluation of correctness of the solution.

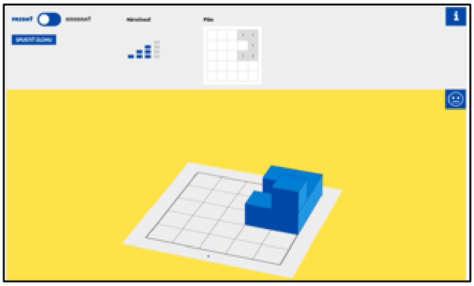

Figure 2. Construct a building from the plan

Source: Own work

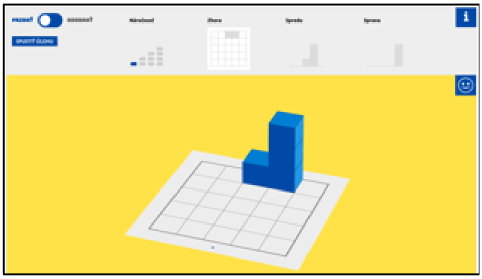

Figure 3. Create a base plan of the building

Source: Own work

Applets No. 3 and No. 4 generate inverse tasks with a focus on development of ability to understand three views (Figure 4, Figure 5) and to be able to apply them. Three view do not have to be a unique record of a building from cubes. Hence, the evaluation of correctness of the solution in applet No. 4 was more difficult to implement to the software. We resolved this problem in the way that no correct solution of a pupil was evaluated as correct even when a task could have multiple correct solutions. This property enables the teacher to create good research educational environment for discussion about the number and diversity of solutions within the activities.

In the applet No. 5 a pupil should check the building from cubes according to the three views, find there a mistake and add or remove some cubes to correct this mistake so that all three views correspond with the building. At this time this applet does not have an inverse alternative.

In the design and implementation of the applets we chose to place the building from cubes and its records to the square grid which should help pupils to better orientate themselves. For this reason ~~is~~ correctness is necessary of the creation of the top view or a plan evaluated also in relation to the location of the building from cubes in a square grid. The front view is on the workspace labelled with a small blue dot so that the pupil's orientation after moving the building was easier.

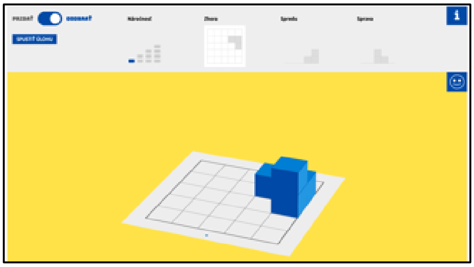

Figure 4. Create a building from three views

Source: Own work

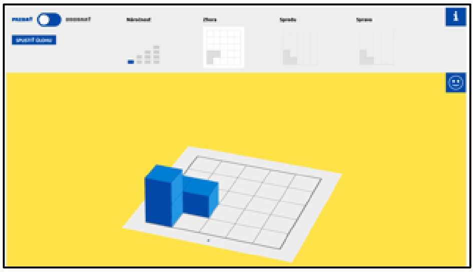

Figure 5. Create the top, front and right view

Source: Own work

From the user's point of view we chose the same design for all types of applets so that the environment of buildings from cubes was known to pupils. On the main workspace (yellow colour) one works with a model of building from cubes, on the help workspace (grey colour) one can change parameters of the task or see plans and three views. Applets enable to add and remove cubes or squares according to the type of task. Adding a cube to the building or a square to the plan is realized only by clicking on the position where the pupil wants to place the object. Feedback about the success of solution of the task is secured via emoticons (happy, sad). Furthermore, in case of correct solution a window with confirmation of success appears. Neither correct nor incorrect solutions are counted. This is because the tasks should not frustrate pupils and not provide space for comparison between pupils from the success point of view. The environment should fulfil the function of free observation and exploration without stress or fear from success.

We plan to insert the information about the use of applets or methodical approach how to use applets to the part labelled with "i". This part will be created after further verification and chosen information will be selected according to the requirements of pupils and teachers. Till now we have been testing intuitive handling with applets and on the basis of results the informational-methodology section will be supplemented. We also think about inserting a short instructional video. In the applets we did not use on purpose many words and we tried to choose control elements so that they were intuitive and could be used without the knowledge of Slovak language. In case it turns out that pupils from other countries have problems with a language barrier we will think about the English version of applets.

Despite the fact that applets are responsive and they should adapt to the device where they are used, we recommend the use of applets on tablets, interactive boards or laptops. Smartphones are not appropriate for children of certain age for manipulation with buildings from cubes, they are too small and do not provide space for mutual sharing, problem solving or working in groups.

5. VERIFICATION AND SWOT ANALYSIS OF APPLETS

In the process of design and implementation of applets we realized cyclical pilot verification with pupils in primary education. They were realized by pedagogy students - future primary education teachers. The aim was to get feedback about how pupils adopt to the new environment for buildings from cubes, whether applets meet their needs, abilities and interests and what their reactions are to the applets and their functionality. Research took place individually with each child and the child was observed while working with the applet. The process was recorded on camera whereas ethic norms of research were complied. From the obtained video-recordings and following transcripts of the videos we extracted codes and categories on the basis of which we made SWOT analysis of applets. From the verification we got information about what children considered to be benefits and disadvantages, what is suitable for them and what is not, what the potential threats are when using applets and so on. We processed results of the video-recordings of 34 children (18 boys, 16 girls) aged 5 to 11 years. The results of the SWOT analysis are described in Tables 2-5.

Strengths of the applets (Table 1) are findings from statements of children and future teachers of primary education about what applets can provide for children from the educational point of view, what benefits they bring, which attributes of applets support interest, needs and abilities of pupils.

Table 1.
Strengths of applets (according to the children and students of pedagogy for primary education)

Strengths
– **Professional and contentual focus of the applet** (development of children's spatial imagination and orientation in the space via buildings from cubes, development of logical and critical thinking, development of mathematical, computer and technical competences)
– **Motivational aspect for a child** (playful and entertaining form, attractive environment, interactivity, natural interest for solving tasks in the applet, learning by playing, children asked for applets to be available at school)

- **Availability and control of applets** (applet is available for free, easy and intuitive handling, understandability of applet, good orientation of the child in the applet's environment, easy manipulation with applet after instructions were given, possibility to rotate building in the applet, good label of the front view – blue dot, easier manipulation with virtual building than with real, applets are appropriate for the interactive board)
- **Methodological aspect of applets** (age appropriate tasks, different levels of complexity of tasks, different types of tasks, possibility to adapt the level of complexity to children's abilities, possibility to correct the solution by adding or removing a cube, understandability of tasks, good orientation using square grid – great help with three views)
- **Feedback** (joy when a smile and message about correct solution appears)

Source: Own research

On the other hand, we observed also weaknesses of applets (Table 2) which means attributes children or future teachers complained about. We also observed that some elements of applets cause negative reactions of children.

Table 2.

Weaknesses of applets (according to the children and students of pedagogy for primary education)

Weaknesses
– **User aspect and control of applet** (instructions needed at the beginning of work with applet, insufficient label of front view – blue dot, impatience when using button add/remove a cube, sometimes children forget about button "start the game" as a start of new game, problem with rotation of model of the building on a smartphone, tablet or 10 inch laptop, size of plan – small numbers in the plan)
– **Instructions for user** (absence of instructions for a child/teacher, methodological instructions for teachers)
– **Feedback** (when a solution is incorrect, the place where is the mistake is not shown)

Source: Own research

Conditions that could help to increase interest about the use of applets were summed up to the category of opportunities (Table 3). Opportunities consist of suggestions for external intervention to the implementation of applet with the aim to improve its professional, methodological or technical aspects.

Table 3.

Opportunities in the use of applets (according to the children and students of pedagogy for primary education)

Opportunities
– **Instructions and methodological instructions** (make and insert a video and audio instructions)
– **User aspect and control of applet** (solve problem with the button add/remove a cube for younger children, some children recommended different colours of cubes, increase the number of cubes that can be placed on each other so it will be possible to build arbitrary building)
– **Feedback as a motivational component** (add also an audio feedback, add points for successful solution, support competitivity and speed)

Source: Own research

When observing children while they were working with applets we identified some attributes that could possibly create threats by usage or could be a possible barrier for use of applets (Table 4). Frequent external threats were caused by choosing an inappropriate level of complexity of applet with regards to needs and abilities of a child or inappropriate methodological process when using the applet.

Table 4.

Threats in the use of applets (according to the children and students of pedagogy for primary education)

Threats
– **Actuality of applets** (development of new software platforms where the applets will not be potentially functional, development of other applets with similar focus)
– **Choice of browsers and devices for functionality of applets** (problem of the browser Internet Explorer, in some browsers the square grid does not show, smartphones are not appropriate although the applets are functional on them, it is necessary to move the screen on some devices so that the whole workplace will be visible)
– **Demotivation and disinterest of children** (panic and disinterest to solve tasks when bad level of complexity was chosen, tiredness, distractions, impatience when an applet is used for longer time, frequent mistakes causing anger, disinterest when inappropriate device with regards to age and motorics of child is chosen)

Source: Own research

The SWOT analysis showed a need to add methodological-informational part to the applet. This part will contain not only information about the usage of applets but also methodological instructions for teachers and video instructions. A discussion about the change in feedback or points for correct/incorrect answers will take place in the future. The result of this discussion will depend on the goal of applets. Verification showed important impact of a teacher on the process of working with applet by the child. Motivation and interest of a child was supported in case when the teacher chose appropriate order and complexity of tasks. On the other hand, if the children’s abilities were overestimated or the time of usage of the applet was too long children showed signs of impatience, distraction, anger or aggression. Therefore, we recommend to combine usage of applets with virtual buildings from cubes with constructing real models of buildings, diversify the activities and strictly set the time for work with applets.

CONCLUSION

We presented geometric applets as a supporting tool for understanding of spatial geometric figures and development of spatial imagination by elementary school pupils. It is important according to Žilinskiene, Demirbilek (2015) that educational mathematical applets in education are clear and user friendly. Another important aspect is the possibility of using materials in mother tongue and also the possibility of creating simple outputs. Next important factors were the possibility of accessing the software through the internet and the fact that the educational software has animations, pictures, multimedia elements. We developed our educational geometric applets that fulfilled the mentioned conditions.

Our work with the applets shows that motivation and interest of pupils was supported in case the teacher chose an appropriate way by giving tasks to pupils. On the other hand, if the pupils obtain too difficult tasks or the time of usage of the applet was too long, then they showed signs of impatience, distraction, anger or aggression. Therefore, we recommend to change activities. After usage of applets with virtual buildings from cubes, it is advisable to work with real models of buildings and also change different kinds of activities in an appropriate way.

It is needed the experimental evaluation of effectivity of above described applets for the development of pupils’ spatial imagination in elementary school. For this reason, we expect during the next school year the realization of the educational experiment in the elementary school - years 1 to 3 concerned with the development of geometrical thinking through educational web pages (see Partová, Žilková, 2017 a, b, c, d, and e).

The experiment will be conducted in a Polish elementary school in Silesia Province. The research will be carried out using the pedagogical experiment method in two groups. They will include elementary, general, and non-profiled children. The SWOT analysis presented in this article will be used for preparing future research on the use of the presented educational tools for supporting geometrical thinking by pupils on the elementary level (see also Rostkowska, 2017).

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