



AUTOMATION OF MATHEMATICAL KNOWLEDGE CONTROL WITHIN DYNAMIC MATHEMATICS PROGRAMS

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Abstract: *It has been established that there are computer tools (DMP – dynamic mathematics programs MathKit, GeoGebra, etc.), through which a computer-aided, but not formal check of mathematical knowledge is possible. The authors highlight the ways of automating mathematical knowledge control on the basis of DMP*

and discuss the experimentally proved hypothesis about the statistical equation of time required for the execution of geometrical tasks for construction, for research and for the geometric point of points. It has been established that for implementation of any form of control (traditional written or performed on the basis of DMP) the subjects of studying must allow the same amount of time. It has been proved that the form of control substantially affects the distribution of students by levels of educational achievements, in particular, statistically lower in the computer verification of knowledge based on DMP.

Keywords: dynamic mathematics programs; e-learning; digital technology; control automation; mathematical knowledge.

INTRODUCTION

The spread of information technology to all sectors of the functioning of society has led to the emergence of computer tools to support the educational process. This contributed to the use of a package of office programs (texts, presentations, etc.), the attraction of specialized subject-oriented software, as well as the creation of computer programs for knowledge control. The last one, as a rule, is aimed

at testing as a method of diagnosing educational achievements, which involves in most closed forms of response (one of many, several of many, establishing conformity, ordering, etc.). However, testing cannot always characterize the actual state of mastering the educational material. This is especially true of mathematics as a field of knowledge, for which the logic of reasoning is often more important, their validity and conciseness, rather than the answer.

From these positions testing as a form of mathematical knowledge control is not always effective, and therefore, computer tools are becoming more in demand which, on the one hand, simplifies the process of control for a teacher who has a sufficient level of formed digital competence, and on the other hand, they monitor the correctness of solving the tasks set.

1. ANALYSIS OF CURRENT RESEARCH

In line with the renewed recommendations of the European Parliament and the Council of the EU, digital competence includes a confident, critical and responsible use and engagement with digital technologies for learning, work and life in society and identified as one of the key to lifelong learning. (Council Recommendation, 2018). In addition, the significance of ICT technologies, digital competence is outlined in a number of regulatory documents, among them are: programs «UNESCO ICT competency framework for teachers» (UNESCO ICT competency, 2011), «European Framework for the Digital Competence of Educators» (DigCompEdu, 2017), etc.

The issue of computer control of knowledge is the subject matter of research carried out by a number of scholars. Thus, N. Morze and V. Vember have investigated the implementation of peer evaluation in the educational process (Morze, et al., 2019). V. Bykov and M. Shyshkina have discovered the possibilities of cloud-oriented technologies for evaluating the educational process (Bykov, et al., 2016). O. Kolhatin and L. Kolhatin have examined the quality of testing procedures and the interpretation of test results in the information and communication pedagogical environment (Kolhatin, et al., 2013). There are also studies on the control of mathematical knowledge, including through ICT, mathematical methods, etc. (Sontag, 2013; Bellman, 2016; B. Craven, 2012, etc.). These studies set out the theoretical and methodological foundations for controlling mathematical knowledge, outlining the historical origins of the formation of forms and methods of control in the study of mathematical disciplines.

We also consider the results of E. Smyrnova-Trybulska, who has substantiated the importance of digital technologies and e-learning in order to improve the quality of the educational process (Smyrnova-Trybulska, 2018). We consider fundamental in the context of the subject matter of our work the results of research which outlines the leading directions of application of information

and communication technologies in the educational process (Bain, et al., 2010; Cardos, et al., 2009; Kostolanyova, 2013, etc.). We also find interesting the results of research on the use of dynamic mathematics programs in the educational process. Thus, Y. Zengina et al. demonstrated the effects of dynamic mathematics software GeoGebra on student achievement in trigonometry teaching (Zengina, et al., 2012). B. Güven et al. Presented a study on the effect of dynamic geometry software (DGS) Cabri 3D on student mathematics teachers' spatial skills was examined (Güven, et al., 2008). In the research by V. Mudaly et al. emphasis is placed on the fact that new methods of teaching mathematics are being sought with the purpose to improve teaching and learning while making mathematics relatable to the new generation of learners. In addition, the authors investigated effectiveness of the use of the GeoGebra app and found that it lies in allowing learners to successfully discover the properties of straight line graphs (Mudaly, et al., 2019).

The results of research confirm that computer testing does not always allow demonstrating the attitudes of a test taker, and fixes only the result of training. It has also been confirmed that the organization of qualitative control over academic achievements requires a considerable amount of time and, sometimes, the involvement of experts in the field of standardization, psychology, pedagogy in order to provide an adequate assessment based on tests, validity of tests, compliance with tests for age characteristics, etc.

The study of the peculiarities of the test control of academic achievements in the field of mathematics revealed the active use of computer testing programs in secondary education institutions. At the same time, scientists are not only updating the problem of developing test tasks, but also the need to create such computer tools, where it would be possible to follow the logic of reasoning by subjects of learning.

The analysis of the programmatic tools of the subject (mathematical) direction was revealed by the class of dynamic mathematics programs (DMP, which include MathKit, GeoGebra, Cabri, The Geometres SketchPad, Gran, etc.), which provides the possibility of studying / researching individual properties or numerical characteristics of mathematical objects based on the results of its direct operation (Semenikhina, et al., 2017). The request of educators to automate the control of mathematical knowledge led to the development of these programs in the direction of expanding their methodological tools. The latest versions of individual dynamic mathematics programs (DMP) have been replenished with additional computer tools, the use of which is not limited to simple testing, and at the same time can facilitate the organization of control over academic achievements precisely in the field of mathematics. These tools and ways of their use in the educational process are the subject of our study.

The aim of the study is to clarify the ways of automating mathematical knowledge control in dynamic mathematics programs and to analyse their effectiveness.

The aim led to the following tasks: 1) to substantiate the possibility of using DMP for automation of mathematical knowledge control; 2) to clarify the ways of using separate DMP for automation of mathematical knowledge control; 3) experimentally confirm the efficiency of the specified ways of using separate DMP for automating the mathematical knowledge control.

2. MATERIALS AND METHODS

A set of methods was used for solving the tasks: *theoretical* – the analysis of scientific and pedagogical sources and computer tools of the DMP to substantiate the possibility of their use for automation of mathematical knowledge control, generalization of methodological approaches and *modelling* of pedagogical situations for determining the ways of using DMP for the organization of control; *empirical* – *surveys*, *observation* of educational activities, pedagogical experiment to confirm the effectiveness of attracting PDM for automation of mathematical knowledge control; *statistical* – Student's average scoring criterion for comparing time spent on traditional and automated forms of control, and McNamara's criterion for determining the influence of the form of mathematical knowledge control on the distribution of learning outcomes.

The experimental base of the study became the Makarenko Sumy State Pedagogical University. Examination of test materials was carried out by leading teachers of the Borys Grinchenko Kyiv University.

The study was conducted in three stages. The first stage envisaged the study of scientific and methodological works of leading scientists in order to justify the possibility of using DMP for automating the mathematical knowledge control. The second stage was practice-oriented, since it required the acquisition of experience in working with DMP to determine the ways of using separate tools of DMP to automate the mathematical knowledge control. At this stage an examination of the tasks used during the pedagogical experiment to control academic achievements was conducted. The third stage was aimed at statistically confirming the effectiveness of attracting DMP for automation of mathematical knowledge control.

3. MAIN RESULTS

V. Proshkin and O. Semenikhina research results on the use of computer mathematical tools in the process of professional training of future mathematics teachers (Proshkin, et al., 2018) allowed determination of the theoretical basis of the use of information tools for controlling academic achievements, in particular the involvement of DMP for automation of mathematical knowledge control.

Our analysis of computer tools of DMP MathKit, GeoGebra (the programs selected as the most popular according to the survey of future teachers of mathematics

and working teachers) made it possible to determine the ways of automating mathematical knowledge control, among which are:

- 1) direct verification of the integrity of the design;
- 2) step-by-step demonstration of the solution;
- 3) the use of special control tools (*Check Response* for automatically checking the answer through a pre-implemented solution algorithm, *Response Field* for open-form questions, a *Checkbox* for closed-form question answers (with one or more correct answers)).

Statistical analysis of the results of the pedagogical experiment found that the form of control significantly affects the distribution of students by levels of academic achievement.

3.1. Discussion

Describe in more detail ways the process of automation of mathematical knowledge control on the basis of DMP and conducted pedagogical experiment.

1. Direct examination of the integrity of the design.

Using DMP it is possible to check the correctness of the construction through the interactive effect on the object – the change in the position of the elements on which the construction is constructed, should not affect the correctness of the reflection of the result.

Quite often, when constructing a mathematical model of a task, the subjects of study depend on the visual similarity of the geometric design, rather than the established rules of construction. In this case, even with minor changes in the position of the base objects the integrity of the structures is violated.

Example 1. Build a direct of Euler (GeoGebra, Figure 1).

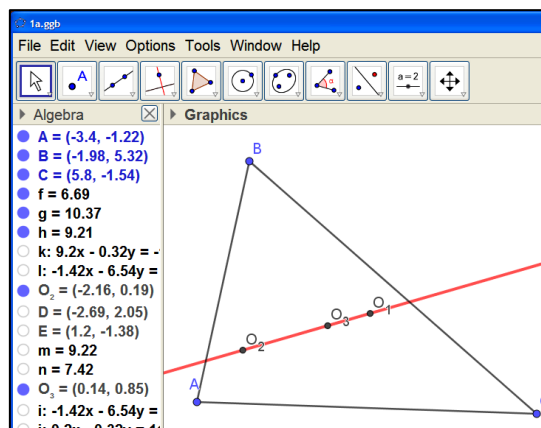


Figure 1, a. Correct construction

Source: Own work

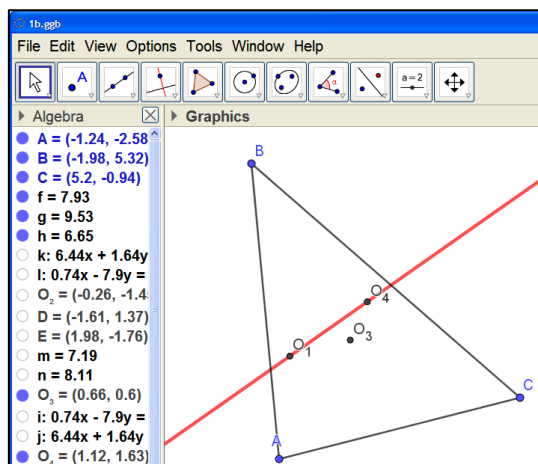


Figure 1, b. Incorrect construction

Source: Own work

Direct of Euler is a straight line on which there are three centres (centroid is a point O_1 , orthocentre is a point O_2 and the centre of the circle described is a point O_3) of any triangle.

The described method of control is fast, but is used as a rule when checking geometric tasks (tasks for construction, problems on the geometric point of points, problems on the construction of sections of polyhedra).

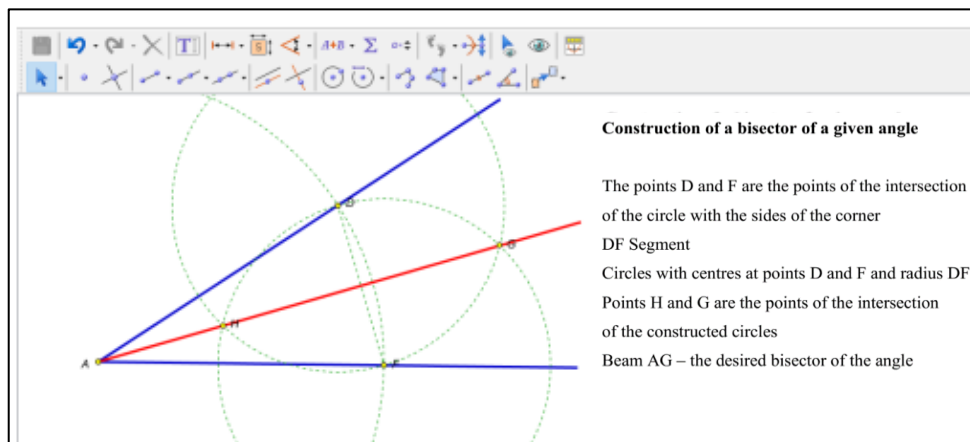


Figure 2, a. A step-by-step demonstration of constructing a bisector of a corner in MathKit

Source: Own work

2. Step-by-step demonstration of the solution.

DMP provides for a step-by-step demonstration of the solution. So, for MathKit, a turn-based demo is set by the *Show / Hide* and *Presentation* buttons (Figure 2, a).

You can also use buttons for *GeoGebra* for a step-by-step demonstration, but for this you need to write a script of the buttons in the Java language or go to the *Steps to build* or activate the *View / Protocol*, where the mode *Steps to build* is also active (Figure 2, b).

Example 2. Build a bisector of a given angle.

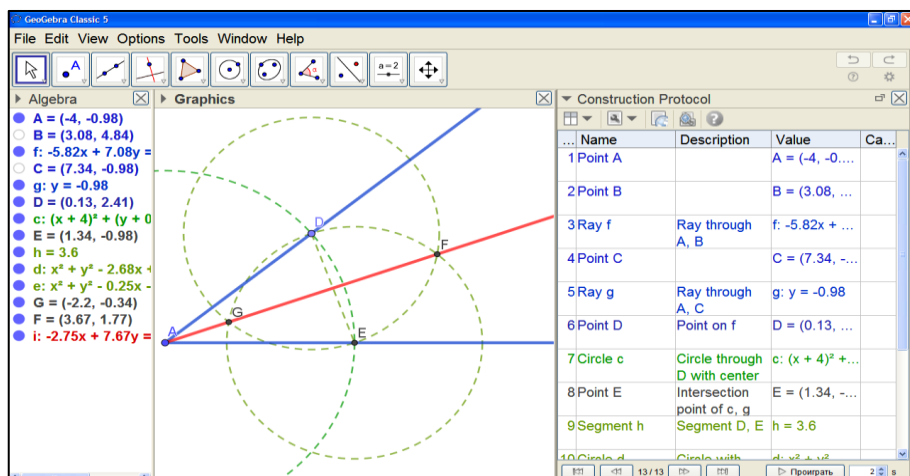


Figure 2, b. A step-by-step demonstration of building a bisector of a corner in GeoGebra

Source: Own work

In this way, we have the opportunity to check the logic of the subjects of the study in solving a mathematical problem, but we consider this form of control to be partially automated, since its use requires additional time expenditures to review each step of the solution and to analyse their correctness.

3. Use of special control tools.

The developers of the MathKit program offered *Checkbox*, *Response Field*, *Check Response*.

The *Checkbox* tool is intended to automate the test check of knowledge on a two-point scale "correctly-wrong". To use the *Checkbox* tool, you create fields where you can write the text of the terms of the task and place a mark-the choice of the correct answer.

To organize the checking of knowledge on the basis of checkboxes using the tool *Check Response*: if the options for the subject of the textbook fully coincide with the reference, the message is displayed on the correct answer.

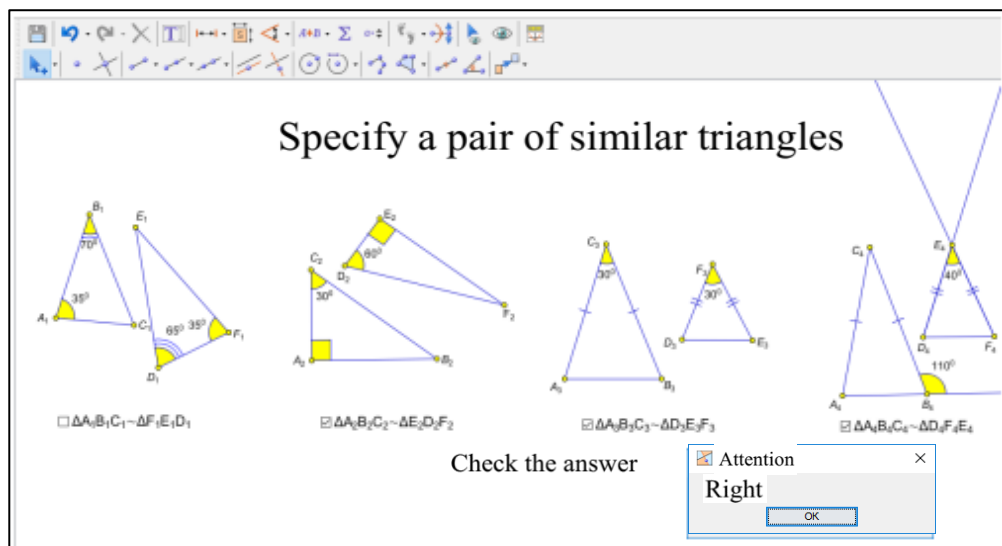


Figure 3. Specify a pair of similar triangles

Source: Own work

Example 3. Specify a pair of similar triangles (Figure 3).

Another tool – *Response Field*, allows you to create a reply box that is inputted from the keyboard. Such a tool is an analogue of the test with an open form of response. When using the tool to create a test task on the screen, a field for inputting a result and a window of properties of the button will appear at the same time, where instead of red text it is necessary to write the correct answer variants (by default their number is 3, and this number can be increased).

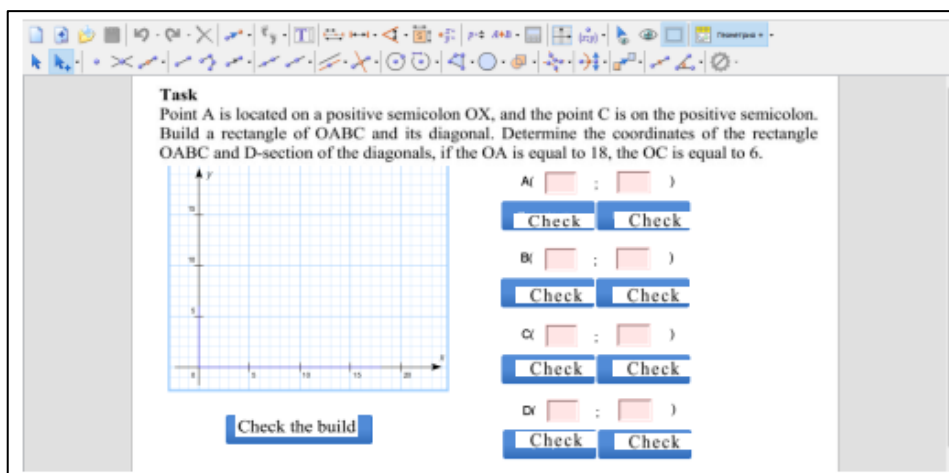


Figure 4. Organizing a response check at MathKit

Source: Own work

Example 4. Point A is located on the positive part of the abscissa, and the point C is located on the positive part of the ordinate axis. Build a rectangle $OABC$ and its diagonals. Determine the coordinates of the vertices of the rectangle $OABC$ and point D in the intersection of diagonals, if the length of the side of the OA is 18, and the length of the OC side is equal to 6.

Technically, the creation of such a type of task for the organization of control is not complicated, but requires careful consideration to take into account all possible options for entering the answer – the order of numbers, the format of numbers, the register of letters, the use of punctuation marks, etc. (Figure 4).

Using the *Check Response* software tool, MathKit allows you to automate the verification of the logic of reasoning when solving a mathematical problem, which is not provided in other software tools of mathematical direction. To organize automated control of mathematical knowledge, you need to build or calculate,

then select objects that are the answer to the task, and lock the *Check Response* button. After that, all intermediate builds and results are hidden, but only the condition remains and the button which was created. Note that the program developers have the ability to edit scripts using the buttons.

Teacher offering to solve a task in the program MathKit, which provides a button *Check Response*, can immediately check the correct answer and do not waste time understanding the method of solution, which may be not one.

Example 5. Build an angle of sinus equal to $3/5$ (Figure 5).

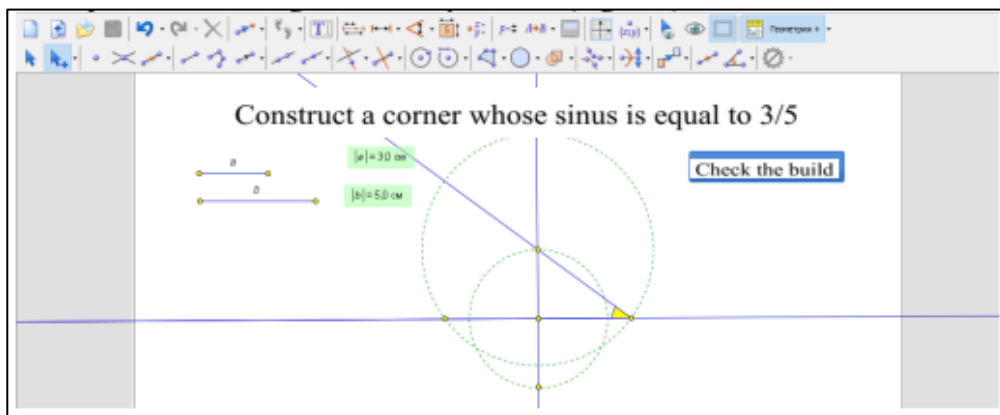


Figure 5. Automated validation of construction in MathKit

Source: Own work

The study of the tool confirmed its correct work under the condition that the object of verification is a point, line, segment, etc. (basic geometric object). For more details about the correctness of this tool we noted in our previous study (Semenikhina, 2014).

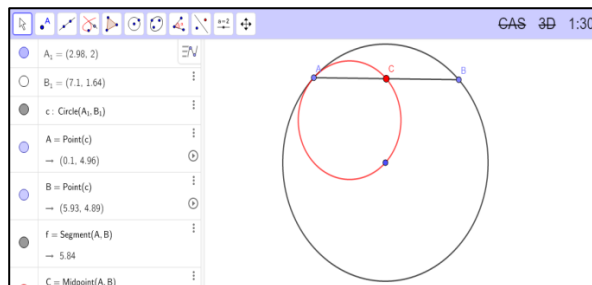


Figure 6. GeoGebra Exam Mode

Source: Own work

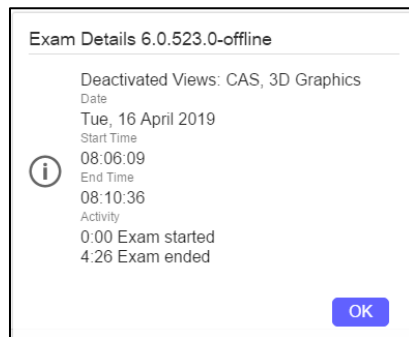


Figure 7. Journal of the task in GeoGebra Exam Mode

Source: Own work

In order to increase the independence *GeoGebra* developers provide *GeoGebra Exam* mode for restricting access to certain computer tools (selected by the teacher) and files hosted on a computer, as well as a ban on access to the Internet (Figure 6). The mode does not automatically control the learning achievements, but fixing actions in a special journal confirms / refutes the independence of the tasks. The log records: the date and time of the beginning of the task execution, the settings set, the exit from the full screen mode, if any, and the return to it, the time of the shutdown.

At the end of the exam, you can see the details of his passage in the journal (Figure 7) – the task is unlocked for 4.26 minutes.

The results of the study were subjected to statistical analysis. We examined the feasibility of using the described ways of automating control from two positions: the average score for comparing the time spent on traditional and automated forms of control (Student's criterion) and whether the chosen form of mathematical knowledge control on the distribution of learning outcomes influences (McNamara's criterion with the scale "Passed / Unpassed").

Describe the results below.

According to the same themes of the special course "Using a computer in teaching mathematics", evaluation of educational achievements was carried out in two different forms: the first provided for the usual solving of the tasks of the topic on the sheets without detailed explanations (Figure 8), the second is the use of DMP for automation of control without detailed explanation (Figure 9). The experts confirmed the identity of the test tasks and their compliance with the course work program, while it was confirmed that it was not possible in the first version to follow the logic of the considerations of those who were tested, unless the construction plan was required to be reproduced.

Table 1.**Statistical processing of experimental results**

Topic	AVG Runtime control works on paper	AVG Runtime tests using DMP
Tasks for construction	18.2 min	20.1 min
Tasks for research	20.3 min	17.6 min
Tasks on geometric place of point	15.3 min	13.9 min

Source: Own work

The hypothesis of statistical equality of averages (Student's criterion): accepted at the level of significance 0,05. The calculations confirm that the average time spent by students on the tasks is statistically the same ($T_{critical}=1,97 > T_{empirical}=1,29$ for construction tasks, $T_{critical}=1,97 > T_{empirical}=1,56$ for research tasks, $T_{critical}=1,97 > T_{empirical}=1,42$ for tasks on geometric place of point).

At the same time, the analysis of the results in two forms of control revealed two facts:

- 1) verification of mathematical knowledge in the second form (using DMP) was faster;
- 2) verification of mathematical knowledge in the second form gave less number of successful assessments, which is statistically confirmed by the McNamara's criterion (Table 2.).

Table 2.**Statistical processing of experimental results**

H ₀ : The form of control does not affect the distribution of academic achievements of students				Double-sided McNamara's criterion for n>20 (n=B+C)
H ₁ : The distribution of students in terms of their academic achievement depends on the form of control				
Results (DMP)				
Topic	Written results	Passed	Unpassed	T _{critical} =3,84 T _{experimental} = =(b-c)^2/(b+c)
Tasks for construction	Passed	A=49	B=16	T _{experimental} =5,76> T _{critical}
	Unpassed	C=5	D=12	Accepted H ₁

Tasks for research	Passed	A=47	B=15	$T_{\text{experimental}}=3,86 > T_{\text{critical}}$
	Unpassed	C=6	D=14	Accepted H_1
Tasks on geometric place of point	Passed	A=42	B=18	$T_{\text{experimental}}=6,54 > T_{\text{critical}}$
	Unpassed	C=4	D=18	Accepted H_1

Table 2 uses the notation: H_0 – zero hypothesis, H_1 – alternative hypothesis.

Source: Own work

This means that the form of control significantly affects the distribution of students by the levels of academic achievement – when performing written work, student success is higher, rather than in the computer verification of knowledge based on DMP.

CONCLUSIONS

The conducted research leads to the following conclusions.

1. According to the results of scientific research the possibility of using computer technologies in the organization of control has been substantiated. In particular, it has been found that there are computer tools that allow automated but not formal testing of mathematical knowledge. These tools include mathematics programs MathKit (or Mathematical Designer) and GeoGebra.
2. Among the ways of automating the mathematical knowledge control on the basis of the indicated DMP the following are defined: direct (interactive) verification of the integrity of the design; step-by-step demonstration (reproduction) solution; the use of special control tools such as Checkbox, Response Field, Check Response, and the use of special modes such as GeoGebraExam.
3. According to the results of the statistical analysis (Student's criterion), the hypothesis of the statistical equality of the time required for the execution of geometrical tasks for construction, for research and on geometric place of point. This means that any form of control (traditional writing or on the basis of the DMP) requires the subjects of the study to have the same amount of time for their implementation. At the same time (McNamara's criterion), that the form of control substantially affects the distribution of students by the levels of academic achievement – when performing control works, the success of students varies. In particular, it is statistically lower in computer testing of knowledge based on DMP.

According to the results of the pedagogical experiment, we consider it expedient to introduce automated forms of control of mathematical knowledge based on the use of the programs MathKit and GeoGebra, without isolating at the same time some of the ways of its implementation.

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