



UTILIZATION OF MATHEMATICAL SOFTWARE IN FAVOUR OF TUTORING PROCESSES

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Abstract: *Maths Support Centres are becoming increasingly importance in the Czech Republic. The existence of these centres can have a positive influence on the mathematical preparation of students of engineering. Also, students of other study fields can utilize these services for professional development. As the suitable interconnection with educational procedures, the information technologies can be considered. In the context of using the information technologies, several teaching techniques were proposed in the form of ideas, which can be applied in favour of efficiency of the tutoring processes. Some of them would be e.g. based on the adaptive feedback strategy with regards to the pedagogical cybernetical approaches. In this paper, the possibilities of mathematical software are presented with a view to utilising them in the feedback process of tutoring in the Maths Support Centre at Tomas Bata University in Zlin. In the presented proposals, the geometry was selected as the most frequently appeared and favourite topic obtained in the frame of the previous quantitative analysis of students' preferences.*

Keywords: Maths Support Centre, pedagogical cybernetics, feedback strategy, mathematical software, geometry, GeoGebra, quantitative analysis.

INTRODUCTION

In the frame of a preparation of the Czech university students, the Maths Support Centres (Patikova, 2016; Zidek & Kotulek, 2019) have a significant role. As the main aim, the efficiency of understanding mathematical topics and building teaching strat-

egies can be considered. There exist many teaching strategies based on an individual approach of each tutor, because the academical experiences can bring to the tutoring the students the original approach. In this strategy, the including the information technologies can enrich the feedback process (Cevik et al., 2015) in the tutoring. In this paper, the extension in the form of the software utilization is practically presented.

In the Czech Republic, the network of the existing Maths Support Centres has an extending trend. The first centre was established as the Maths and Stats Support Centre by the head of the Department of Applied Mathematics and Computer Science – Assoc. Prof. Maria Kralova (the author e.g. of publication: Kralova & Lasak, 2018) from the Faculty of Economics and Administration of the Masaryk University. The second Maths Support Centre was built by the Department of Mathematics by Dr. Zuzana Patikova (Patikova, 2016) under the guarantee of the Faculty of Applied Informatics of the Tomas Bata University in Zlin. The third centre is located at Department of Mathematics and Descriptive Geometry at the VSB – Technical University of Ostrava with the leadership of Dr. Jan Kotulek (Zidek & Kotulek, 2019). The fourth centre is being prepared at Faculty of Education of the Palacky University.

In each centre, a structure of the provided topics depends on the particular study fields of the departments. There are appeared topics based on the pure and applied mathematics focusing the mathematical analysis, algebra, geometry or statistics. The problems of economics and also control engineering practice can be seen as the examples of a wide spectrum of various study fields.

The classical teaching strategies applied by academics in the frame of the tutoring can be enriched by procedures with aim of providing the fast help and a feedback to student in the centre. Using the support in the form of information technologies can be suitable implemented in this process. In the article (Barot, 2017), the proposal of the adaptive control strategy was presented for purposes of application in the maths support centres. Generating the computer based mathematical examples was described in (Barot, 2019). Both proposals were the extension with regards pedagogical cybernetics (Granic et al., 2009; Gushchin & Divakova, 2015). In pedagogical cybernetics, the similar concepts of feedback principles are considered in favour of an achievement of the efficiency and appropriate control quality of realized processes. Tutoring the students can be suitably classified as a part of the feedback procedure in which information technologies can be helpful.

A simplification of the explanation steps can be appropriate in case of a situation when one tutor provides information for a number of visiting students in a group. On the previous proposal (Barot, 2017), based on general recommendation of the free-available software or e.g. some simplifications in approaches of solving the integrals (Barot, 2018), this paper concretely presents possibilities for supporting the geometrical discipline in the centres.

The topic of geometry was preferred by students of the Department of mathematics with Didactics at Faculty of Education, University of Ostrava in the realized quantitative research with 1100 proposals of the software utilizations in mathematics. At second, the analysis also confirms the statistical significance of the students approaches based on selection of mathematical fields, topics, and problems. The utiliza-

tion of the geometrical software or the augmented reality has been frequently used in the education, e.g. in (Kmetova et al., 2019; Lavicza & Papp-Varga, 2010). This type of application of information technologies can be generally seen in the field of the STEM education, e.g. in (Hott & Dibbs, 2020). The significant important freeware solution for these applied purposes is GeoGebra (Korenova, 2017). A wide spectrum of types of the geometrical models have been proposed by the GeoGebra community. Therefore, the geometrical modelling belongs to possibilities of the modern information technologies which have proposition to be a part of the feedback educational process. Therefore, the utilization can be a suitable tool in the realization of the tutoring the students in the maths support centre. In this paper, possibilities of the GeoGebra's solution in favour of the tutoring are described.

1. APPLIED ADAPTIVE FEEDBACK CONTROL STRATEGY WITH REGARDS TO TUTORING PROCESSES

With regards to the general principles of cybernetics, pedagogical cybernetics (Granic et al., 2009; Gushchin & Divakova, 2015) have the same principles as well as technical cybernetics frequently does within the control system theory (Corriou, 2004). The same background is built on the feedback strategy (Cevik et al., 2015) in the frame of control some process or processes. Therefore, each educational process, which fulfils the principle of the feedback control, can be seen as a part of cybernetics. Due to a wide spectrum of individual behaviour of students, the modelling can have a significant role in the feedback process. Extended proposals of strategies inspired by the cybernetical disciplines can be seen e.g. in adaptive control strategy used in educational conditions (Barot, 2019). Moreover, the cybernetical principles can be bound to the statistical modelling with utilization of the cybernetical mathematical models in the quantitative research, as discussed by e.g. Vaclavik et al. (2019).

In the adaptation approach, the teacher is situated in the role of the controller and the student as the controlled object in the frame of the feedback control with the re-identification. The re-identification is based on the continuously based exploration of the student's results. This step can be supported by the using the information technologies (Barot, 2017).

As a particular procedure, using the generator of random numbers in a construction of systems of the linear equations (Barot, 2019), simplification of solving the partial fractions in computing the integrals (Barot, 2018) or the general role of the free-available software (Barot, 2017) can be practically used with complementing the professional experiences of each member in the maths support centre. For applications of proposals in the frame of pedagogical cybernetics, the math support centre fulfils the assumption for their useful realization.

2. DESIGN OF QUANTITATIVE ANALYSIS OF PREFERRED MATHEMATICAL SOFTWARE OPTION

For the particular proposals of the study tools using the mathematical software, the opinion of students about the preferred software solution was analysed and identified through quantitative research, with its structure and aims described in this session.

In the section of results, only the most preferred software possibility is therefore used for the designed tools for purposes of the tutoring process for the Maths Support Centre in Zlin, where the wide spectrum of mathematical topics has been considered e.g. for study fields of automation and control, management and economics, security technologies, computer sciences, information technologies.

The opinions of students were analysed in the frame of the defined task. Task was consisted of declaration of 51–100 own defined mathematical examples with the analytical solution and the solution occurred in the own selected mathematical software tool. The respondents were the students at the Department of Mathematics with Didactics at the Faculty of Education at University of Ostrava. Between both institutions, the applied research of the educational strategies has been realized and published yet e.g. (Barot, 2017). The main idea is the connection of the development of the new or modified educational proposals with the practical application in the environment of the Math Support Centre.

There has been existed a wide spectrum of solutions based on numerical or symbolic computational core of software. The most frequently required topics can be bound on the university mathematics e.g. linear algebra, geometry, mathematical analysis, functions, combinatorics, statistics and probability, arithmetic. E.g. in most popular category of the STEM education (Hott & Dibbs, 2020), the geometrical modelling holds the significant position with regards the improving student's stereometrical imaginations. GeoGebra software solution can be considered as the important geometrical tool used in the field of the education, as can be seen e.g. in the quantitative study by Lavicza and Papp-Varga (2010).

The main aim is the selection of the most preferred option of using the software solution according to the students' practical choices in their task (51–100 own defined mathematical examples). However, the following research question confirmation is necessary to accept the statistical dominance of the obtained option (the rejection of homogeneity of frequencies in contingency tables). There are considered the following research questions RQ1–RQ4 bound with the declaration of the statistical hypotheses $1H-4H$.

RQ1: Are there statistically significant differences between the study field combinations and the selection of particular mathematical software options?

RQ2: Are there statistically significant differences between the mathematical field and the selection of particular mathematical software options?

RQ3: Are there statistically significant differences between the mathematical topics and the study field combinations?

RQ4: Are there statistically significant differences between the mathematical topics and the selection of particular mathematical software options?

All these defined hypotheses will be tested on the significance level, as is assumed in the methods of the mathematical induction, which belong to the quantitative approaches (Gauthier & Hawley, 2015). The statistical significance has the important role in the applied quantitative research in the social sciences, e.g. (Lackova, 2014; Simbartl & Honzikova, 2016; Polasek & Javorcik, 2019) and is carrier in the engineering applications (e.g. Barot et al., 2020 in Print).

In the present quantitative research, the observed parameters will be divided into several categorical variables for each record of each students' example: mathematical software, mathematical field, mathematical topic, study field of the student. Due to the character of these considered categorical variables, the Chi-Squared test (Gauthier & Hawley, 2015) will be considered for the testing the hypotheses.

1H: $1H_0$ (in $1H_1$ is missing "not"): There are (not) statistically significant differences between the study field combinations and the selection of particular mathematical software options on the strictly defined significance level of 0.001.

2H: $2H_0$ (in $2H_1$ is missing "not"): There are (not) statistically significant differences between the mathematical fields and the selection of particular mathematical software options on the strictly defined significance level of 0.001.

3H: $3H_0$ (in $3H_1$ is missing "not"): There are (not) statistically significant differences between the mathematical topics and the study field combinations on the strictly defined significance level of 0.001.

4H: $4H_0$ (in $4H_1$ is missing "not"): There are (not) statistically significant differences between the mathematical topics and the selection of particular mathematical software options on the strictly defined significance level of 0.001.

3. RESULTS

For purposes of Maths Support Centre at the Faculty of Applied Informatics, the following results of the quantitative research influence the type of the proposed mathematical tools. Concretely, the respondents were the students at the Department of Mathematics with Didactics at the Faculty of Education at University of Ostrava, who studied Application of the Computational Technologies in Mathematics during this summer semester in their 1st year of the bachelor study programme. The tutoring processes can be than widely enriched by these mathematical software solutions.

3.1. Students' Preferences Analysed in Quantitative Research

The realized quantitative research contained 1100 records of proposals of 13 students' own definitions of mathematical problems (51–100 examples for each student). The following statistical results were obtained in the following software: MS Excel (statistical description and descriptive plots) and PAST Statistics v. 4 (Hammer et al., 2001) (cluster analysis, testing the normality of data and testing the hypotheses). For the total number of 1100 proposed examples, the average value was 84.69 proposed examples.

For the teaching the mathematics at the 2nd grade of the primary school, the respondents are prepared at the Department of Mathematics with didactics at the Faculty of Education at University of Ostrava. Respondents had a pair of study fields. Mathematics (Mat) is bounded on the 2nd field: Sport (Spo), Informatics (Inf), Technical Education (Tec), English (Eng), Visual Arts (Vis) and Czech Language (Cze).

According to distributions of relative frequencies, 3 mathematical topics were frequently appeared: Algebra with 419 proposals, Geometry with 406 proposals, and Functions with 247 respondents. As the most frequented option, GeoGebra was selected in 472 proposals, i.e. 42.9% of total records.

The wide spectrum of selected spectrum of used mathematical software are at the combination of the study field of Mathematics and Informatics. 192 records of examples (i.e. 17.46%) was computed using the Photomath at combination of the Sport and Mathematics.

With regards to the study field combinations of respondents, the similarities in the utilization of the mathematical software can be seen in Figure 1, where the results of the cluster analysis can be seen. The following interesting similarities were detected: 1st similarity between Inf-Mat and Mat-Inf (focused on informatics), 2nd similarity between Mat-Cze and Mat-Eng (focused on languages), 3rd similarity between Mat-Tec and Mat-Vis (graphical proposals). Two main groups can be seen in the second option of study field with mathematics: informatics (major and minor) and sport (major) in comparison to other study combinations. The cluster analysis was computed with the setting of the Ward method, which has been frequently used for purposes of the analysis of the Euclidean distances between considered data set.

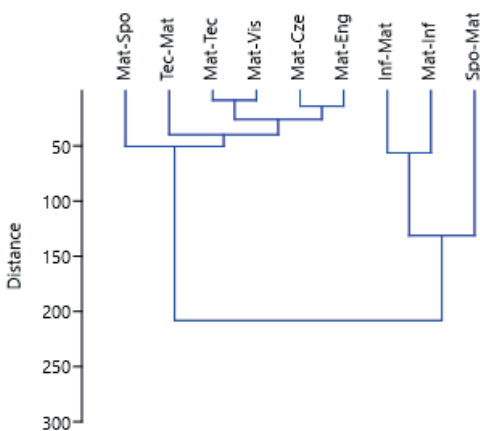


Figure 1. Similarities in Utilization of Mathematical Software Across to Study Field Combinations

Source: Own work.

Using by the Chi-Squared test in the contingency table (occurred proposals for each study field combination according to option of software), the p value was $1.86 \times 10^{-249} < 0.001$. The degrees of freedom were 56 and the Chi-Squared Criterion was 1369. The zero hypothesis $1H_0$ was rejected in favour of the alternative hypothesis $1H_1$. Therefore, there are statistically significant differences between the study field combinations and the selection of particular mathematical software options on the strictly defined significance level of 0.001, as was defined in RQ1.

For reason of unexpecting combination of major topics (Algebra, Geometry and Functions) and the most occurred software possibility of GeoGebra, other mathematical topics instead of the geometry belong to solving the problems using by this software. The students provided the examples of geometry and also functions by the software of GeoGebra. The reason is the advantageously integrated computational system (CAS) into the GeoGebra core.

In the frame of the Chi-Squared test in the following contingency table (occurred frequencies of mathematical topics and the selected software), the p value was $6.69 \times 10^{-306} < 0.001$. The degrees of freedom were 32 and the Chi-Squared Criterion was 1549.2. The zero hypothesis $2H_0$ was rejected in favour of the alternative hypothesis $2H_1$. Therefore, there are statistically significant differences between the mathematical fields and the selection of particular mathematical software options on the strictly defined significance level of 0.001, as was defined in RQ2.

The most particular content of the proposed mathematical problems were geometrical constructions and computations. The study field combinations for this phenomenon were Spo-Mat (with 12.18% of 1100 records) and Mat-Inf (with 9% of 1100 records). Other particular types of problems were distributed uniformly with the lower occurrence. In the frame of the Chi-Squared test in the contingency table (occurred frequencies of mathematical problems and study field combinations), the p value was $4.70 \times 10^{-236} < 0.001$. The degrees of freedom were 80 and the Chi-Squared criterion was 1380.5. The zero hypothesis $3H_0$ was rejected in favour of the alternative hypothesis $3H_1$. Therefore, there are statistically significant differences between the mathematical topics and the study field combinations on the strictly defined significance level of 0.001, as was defined in RQ3.

As can be seen in Figure 2, the most particular content of the proposed mathematical problems was the geometrical construction for the mathematical software GeoGebra (with 31.72% of 1100 records).

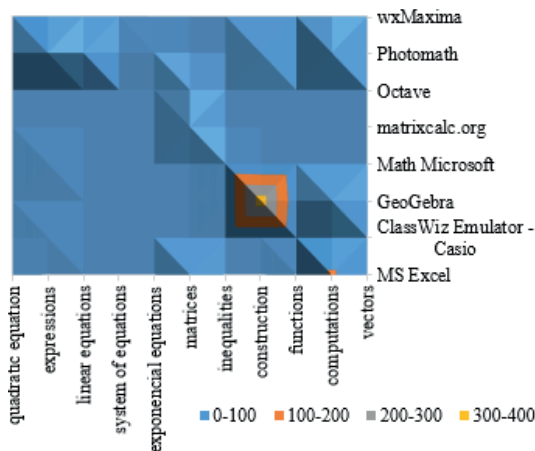


Figure 2. Occurrence of Proposed Mathematical Topics with Regards to Software Options

Source: Own work.

In the frame of the Chi-Squared test in the contingency table (occurred frequencies of mathematical problems and study field combinations), the p value was $0 < 0.001$. The degrees of freedom were 70 and the Chi-Squared Criterion was 2082.7. The zero hypothesis $4H_0$ was rejected in favour of the alternative hypothesis $4H_1$. Therefore, there are statistically significant differences between the mathematical topics and the

selection of particular mathematical software options on the strictly defined significance level of 0.001, as was defined in RQ4.

3.2. Proposals of Geometrical Models in Favour of Utilization in Maths Support Centre

As was proven in the quantitative analysis in the previous section, appearances of p values in each confirmation of research questions expresses, that there are the statistically significant differences between the frequencies in each of the contingency. With the highest value of frequency, therefore the GeoGebra software is considered for the construction of the educational tools for the Maths Support Centre. The following proposals can be advantageously used also by other tutors across the network of the existing Maths Support Centres in the Czech Republic. The topic can be advantageously included in the feedback adaptive control strategy of the educational approach of tutors. These files will be free-available for each student. The sense of these implemented solutions is expected in the form of the improving the efficiency of the tutoring in the field of the geometry, which can be further suitably used also in the problematic mathematical analysis.

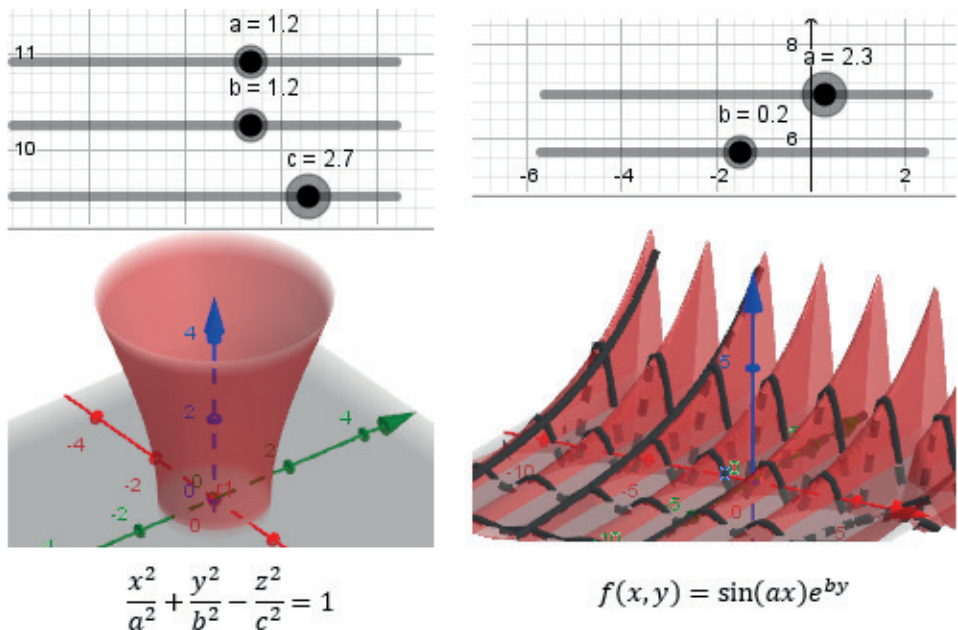


Figure 3. Proposals of Dynamical Models of Hyperboloid and Multidimensional Function

Source: Own work.

On the GeoGebra platform, the geometrical constructions can be advantageously based on the integration of the slider, which can influence the parametrical settings of models. This aspect can be appropriately used as the variables in the analytical description of the considered geometrical objects.

The importance of the analytical geometry can be seen in the mathematical analysis in the frame of the solving the integrals for the multivariable cases, where for the definition area of integrals is the stereometrical imagination the necessary assumption.

With regards to the integrating the parametrization of the analytical expression of geometrical objects, there were constructed dynamical models of: a circle $\{(x - m)^2 + (y - n)^2 = r^2\}$, with parameters $m, n, r\}$, a parabola $\{(y - n)^2 = 2p(x - m)\}$, with parameters $m, n, p\}$, a hyperbola $\{(x - m)^2/a^2 - (y - n)^2/b^2 = 1\}$, with parameters $m, n, a, b\}$, an ellipse $\{(x - m)^2/a^2 + (y - n)^2/b^2 = 1\}$, with parameters $m, n, a, b\}$.

In the three dimensional case, the parametrization of the real function $\{f(x, y) = \sin(ax) \exp(by)\}$, with parameters $a, b\}$ and a circular hyperboloid $\{x^2/a^2 + y^2/b^2 - z^2/c^2 = 1\}$, with parameters $a, b, c\}$ were proposed in the form of the dynamical models in GeoGebra, as can be seen in Figure 3.

The study area of the mathematical analysis was supported by the following models $\{x = \operatorname{argmin}(f(x, y) = x^2 + 4xy + y^2; y \leq ax + b)\}$, with parameters $a, b\}$ (Figure 4), which can enrich the imagination of solving the extremes of the multidimensional functions of the real variables considering the constraints in the form of inequalities, as can be seen also in (Korenova et al., 2020 in Print).

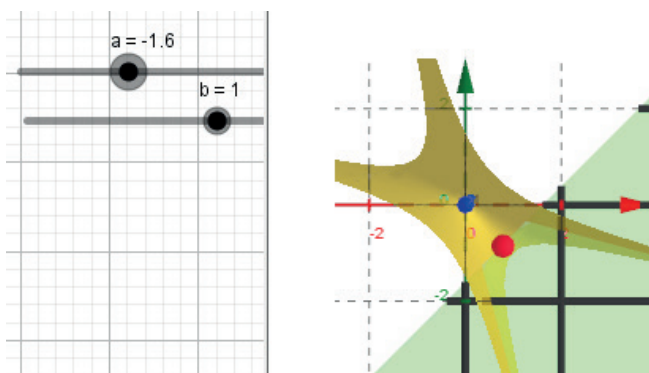


Figure 4. Proposals of Dynamical Models of Constrained Optimization Problems

Source: Own work.

CONCLUSION

In favour of the particular utilization, the proposal of geometrical modelling options was described for purposes of their implementation in the Maths Support Centres. As the obtained option in realized quantitative analysis, the geometrical topic was selected and focused on the utilization of the geometrical modelling. Particularly with regards to statistical significance, differences were identified between the study field combinations and the selection of particular mathematical software, the mathematical fields and the selection of particular mathematical software, the mathematical topics and the study field combinations and between the mathematical topics and the selection of particular mathematical software options.

Opportunities of the application of the geometrical advantages has been widely described in case of educational field yet. However, this presented approach can be suitable in case of application of these principles in the frame of tutoring the students in the Maths Support Centres as a part of the adaptive control teaching strategy. The geometrical modelling can enrich the students' imaginations in the technical fields and can be advantageous for the suitable understanding the following topics of control system theory based on e.g. the optimisation and mathematical analysis, where the geometrical theoretical background has been frequently integrated. Future research can possibly analyse the efficiency of the implementation of practical proposals presented in this contribution.

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